

Contents

Preface	ix
Chapter 1 Finite and Infinite Words	1
1.0 Introduction	1
1.1 Semigroups	1
1.2 Words	3
1.3 Automata	12
1.4 Generating series	22
1.5 Symbolic dynamical systems	26
1.6 Unavoidable sets	33
Problems	40
Notes	44
Chapter 2 Sturmian Words	45
2.0 Introduction	45
2.1 Equivalent definitions	46
2.2 Standard words	63
2.3 Sturmian morphisms	83
Problems	101
Notes	108
Chapter 3 Unavoidable Patterns	111
3.0 Introduction	111
3.1 Definitions and basic properties	112
3.2 Deciding avoidability: the Zimin algorithm	116
3.3 Avoidability on a fixed alphabet	123
Problems	131
Notes	133

Chapter 4	Sesquipowers	135
4.0	Introduction	135
4.1	Bi-ideal sequences	136
4.2	Canonical factorizations	137
4.3	Sesquipowers and recurrence	141
4.4	Extensions of a theorem of Shirshov	144
4.5	Finiteness conditions for semigroups	149
	Problems	159
	Notes	162
Chapter 5	The Plactic Monoid	164
5.0	Introduction	164
5.1	Schensted's algorithm	165
5.2	Greene's invariants and the plactic monoid	167
5.3	The Robinson–Schensted–Knuth correspondence	170
5.4	Schur functions and the Littlewood–Richardson rule	176
5.5	Coplactic operations	182
5.6	Cyclage and canonical embeddings	186
	Problems	192
	Notes	195
Chapter 6	Codes	197
6.0	Introduction	197
6.1	X -factorizations	198
6.2	Defect	204
6.3	More defect	209
6.4	A theorem of Schützenberger	221
	Problems	224
	Notes	227
Chapter 7	Numeration Systems	230
7.0	Introduction	230
7.1	Standard representation of numbers	231
7.2	Beta-expansions	236
7.3	U -representations	248
7.4	Representation of complex numbers	257
	Problems	263
	Notes	267

Chapter 8	Periodicity	269
8.0	Introduction	269
8.1	Periods in a finite word	270
8.2	Local versus global periodicity	283
8.3	Infinite words	294
	Problems	306
	Notes	310
Chapter 9	Centralizers of Noncommutative Series and Polynomials	312
9.0	Introduction	312
9.1	Cohn’s centralizer theorem	313
9.2	Euclidean division and principal right ideals	315
9.3	Integral closure of the centralizer	316
9.4	Homomorphisms into $k[t]$	319
9.5	Bergman’s centralizer theorem	320
9.6	Free subalgebras and the defect theorem	322
9.7	Appendix: some commutative algebra	324
	Notes	328
Chapter 10	Transformations on Words and q-Calculus	330
10.0	Introduction	330
10.1	The q -binomial coefficients	332
10.2	The MacMahon Verfahren	334
10.3	The insertion technique	337
10.4	The (t, q) -factorial generating functions	339
10.5	Words and biwords	341
10.6	Commutations	343
10.7	The two commutations	345
10.8	The main algorithm	347
10.9	The inverse of the algorithm	349
10.10	Statistics on circuits	351
10.11	Statistics on words and equidistribution properties	353
	Problems	355
	Notes	363
Chapter 11	Statistics on Permutations and Words	365
11.0	Introduction	365
11.1	Preliminaries	366
11.2	Words with a given shape	367
11.3	Backsteps of permutations with a given shape	369

11.4	Inversions of permutations with a given shape	372
11.5	Lyndon factorization and cycles of permutations	375
11.6	Major index of permutations with a given cyclic type	378
	Problems	381
	Notes	385
Chapter 12	Makanin's Algorithm	387
12.0	Introduction	387
12.1	Words and word equations	389
12.2	The exponent of periodicity	400
12.3	Boundary equations	406
12.4	Proof of Theorem 12.3.10	432
	Problems	435
	Notes	438
Chapter 13	Independent Systems of Equations	443
13.0	Introduction	443
13.1	Sets and equations	444
13.2	The compactness property	447
13.3	Independence of finite systems of equations	453
13.4	Semigroups without the compactness property	458
13.5	Semigroups with the compactness property	460
	Problems	468
	Notes	470
References	473
Index of Notation	497
General Index	499