Table of Contents

Number Theory ____ 1

1. Six proofs of the infinity of primes 3
2. Bertrand's postulate 7
3. Binomial coefficients are (almost) never powers 13
4. Representing numbers as sums of two squares 17
5. Every finite division ring is a field 23
6. Some irrational numbers 27

Geometry ____ 35

7. Hilbert's third problem: decomposing polyhedra 37
8. Lines in the plane and decompositions of graphs 45
9. The slope problem 51
10. Three applications of Euler's formula 57
11. Cauchy's rigidity theorem 63
12. The problem of the thirteen spheres 67
13. Touching simplices 73
14. Every large point set has an obtuse angle 77
15. Borsuk's conjecture 83

Analysis ____ 89

16. Sets, functions, and the continuum hypothesis 91
17. In praise of inequalities 101
18. A theorem of Pólya on polynomials 109
19. On a lemma of Littlewood and Offord 117

Combinatorics 121

20. Pigeon-hole and double counting 123
21. Three famous theorems on finite sets 135
22. Cayley's formula for the number of trees 141
23. Completing Latin squares 147
23. The Dinitz problem 153

Graph Theory 159

25. Five-coloring plane graphs 161
26. How to guard a museum 165
27. Turán's graph theorem 169
28. Communicating without errors 173
29. Of friends and politicians 183
30. Probability makes counting (sometimes) easy 187

About the Illustrations 196

Index 197