CONTENTS

1

91

I. FILTERED RINGS AND MODULES

Preliminaries	1
The Category of Filtered Modules	2
Remarks	3
Functors on R-filt.	3
Graded Rings and Modules	5
Examples	5
The Category of Graded Modules	5
Some Properties of Graded Modules	7
Tensor Product of Graded Modules	12
Filtration and Associated Gradation	13
Free, Projective and Finitely Generated Objects of R-filt	16
The Functor $HOM_{\mathbb{R}}(-,-)$.	24
Projective Modules and Homological Dimension of Rings.	27
Weak (Flat) Dimension of Filtered Modules.	30
PICS IN GRADED RING THEORY	36
Homogenization	36
The Structure of Principal Graded Rings	38
Noetherian Objects	<u>4</u> 1
Krull Dimension of Graded Rings	47
The Krull Dimension of Some Classes of Rings	53
Polynomial Rings	53
Rings of Formal Power Series	55
The Enveloping Algebra of a Lie Algebra	56
Weyl Algebras	56
Graded Division Rings	58
The Structure of Simple Objects in R-gr.	59
The Jacobson Radical of Graded Rings.	65
Semisimple Graded Rings. Goldie's Theorems for Graded Rings	(7
Graded Matrix Rings	57
Rings of Fractions and Goldie's Theorems	70
Primary Decompositions	74
External Homogenization	78
Graded Rings and Modules of Quotients	83
. Torsion Theories over Graded Rings	83
. Examples of Torsion Theories in R-gr.	89
	The Category of Filtered Modules Remarks Functors on R-filt. Graded Rings and Modules Examples The Category of Graded Modules Some Properties of Graded Modules Tensor Product of Graded Modules Tensor Product of Graded Modules Filtration and Associated Gradation Free, Projective and Finitely Generated Objects of R-filt The Functor HCM _R (-,-). Projective Modules and Homological Dimension of Rings. Weak (Flat) Dimension of Filtered Modules. PICS IN GRADED RING THEORY Homogenization The Structure of Principal Graded Rings Noetherian Objects Krull Dimension of Graded Rings The Krull Dimension of Some Classes of Rings Polynomial Rings Rings of Formal Power Series The Enveloping Algebra of a Lie Algebra Weyl Algebras Graded Division Rings The Structure of Simple Objects in R-gr. The Jacobson Radical of Graded Rings. Semisimple Graded Rings. Goldie's Theorems for Graded Rings Graded Matrix Rings Rings of Fractions and Goldie's Theorems Primary Decompositions External Homogenization Graded Rings and Modules of Quotients Torsion Theories over Graded Rings

12.3. Injective Objects and Torsion Theories

VIII

12.4. Graded Rings and Modules of Quotients	95
12.5. More Properties of Graded Localization	100
II.13. Graded Prime Ideals and the Ore Condition	107
II.14. The Presheaves on Proj R.	110
II.15. Graded Zariski Central Rings	116
15.1. Basic Facts about Zariski Central Rings	116
15.2. GZ- and ZG-Rings	118
II.16. Scheme Structure of Proj over a Graded Zariski Central R	ing 121
III. LOCAL CONDITIONS FOR NOETHERIAN GRADED RINGS	127
III.1. Injective Dimension of Graded Rings	127
III.2. Regular, Gorenstein and Cohen-Macaulay Rings	133
III.3. Graded Rings and M-sequences.	137
REFERENCES	144

SUBJECT INDEX

146