

# Table of Contents

<b>Part I. Classical Theory</b> . . . . .	1
Chapter I. Modular Forms . . . . .	3
§ 1. The Modular Group . . . . .	3
§ 2. Modular Forms . . . . .	5
§ 3. The Modular Function $j$ . . . . .	12
§ 4. Estimates for Cusp Forms . . . . .	12
§ 5. The Mellin Transform . . . . .	14
Chapter II. Hecke Operators . . . . .	16
§ 1. Definitions and Basic Relations . . . . .	16
§ 2. Euler Products . . . . .	21
Chapter III. Petersson Scalar Product . . . . .	24
§ 1. The Riemann Surface $\Gamma \backslash \mathfrak{H}^*$ . . . . .	24
§ 2. Congruence Subgroups . . . . .	29
§ 3. Differential Forms and Modular Forms . . . . .	32
§ 4. The Petersson Scalar Product . . . . .	35
Appendix by D. Zagier. The Eichler–Selberg Trace Formula on $SL_2(\mathbf{Z})$ . . . . .	44
<b>Part II. Periods of Cusp Forms</b> . . . . .	55
Chapter IV. Modular Symbols . . . . .	57
§ 1. Basic Properties . . . . .	57
§ 2. The Manin-Drinfeld Theorem . . . . .	61
§ 3. Hecke Operators and Distributions . . . . .	65
Chapter V. Coefficients and Periods of Cusp Forms on $SL_2(\mathbf{Z})$ . . . . .	68
§ 1. The Periods and Their Integral Relations . . . . .	69
§ 2. The Manin Relations . . . . .	73

§ 3. Action of the Hecke Operators on the Periods . . . . .	76
§ 4. The Homogeneity Theorem . . . . .	81
<b>Chapter VI. The Eichler–Shimura Isomorphism on <math>SL_2(\mathbf{Z})</math> . . . . .</b>	<b>84</b>
§ 1. The Polynomial Representation . . . . .	85
§ 2. The Shimura Product on Differential Forms . . . . .	88
§ 3. The Image of the Period Mapping . . . . .	89
§ 4. Computation of Dimensions . . . . .	93
§ 5. The Map into Cohomology . . . . .	96
<b>Part III. Modular Forms for Congruence Subgroups . . . . .</b>	<b>99</b>
<b>Chapter VII. Higher Levels. . . . .</b>	<b>101</b>
§ 1. The Modular Set and Modular Forms . . . . .	101
§ 2. Hecke Operators . . . . .	105
§ 3. Hecke Operators on $q$ -Expansions . . . . .	108
§ 4. The Matrix Operation . . . . .	111
§ 5. Petersson Product . . . . .	112
§ 6. The Involution . . . . .	114
<b>Chapter VIII. Atkin–Lehner Theory . . . . .</b>	<b>118</b>
§ 1. Changing Levels . . . . .	118
§ 2. Characterization of Primitive Forms . . . . .	122
§ 3. The Structure Theorem . . . . .	123
§ 4. Proof of the Main Theorem . . . . .	126
<b>Chapter IX. The Dedekind Formalism . . . . .</b>	<b>138</b>
§ 1. The Transformation Formalism . . . . .	138
§ 2. Evaluation of the Dedekind Symbol . . . . .	142
<b>Part IV. Congruence Properties and Galois Representations . . . . .</b>	<b>149</b>
<b>Chapter X. Congruences and Reduction mod <math>p</math> . . . . .</b>	<b>151</b>
§ 1. Kummer Congruences . . . . .	151
§ 2. Von Staudt Congruences . . . . .	153
§ 3. $q$ -Expansions . . . . .	154
§ 4. Modular Forms over $\mathbf{Z}[\frac{1}{2}, \frac{1}{3}]$ . . . . .	156
§ 5. Derivatives of Modular Forms . . . . .	159
§ 6. Reduction mod $p$ . . . . .	162
§ 7. Modular Forms mod $p$ , $p \geq 5$ . . . . .	164
§ 8. The Operation of $\theta$ on $\overline{M}$ . . . . .	169

Chapter XI. Galois Representations . . . . .	176
§ 1. Simplicity . . . . .	177
§ 2. Subgroups of $GL_2$ . . . . .	180
§ 3. Applications to Congruences of the Trace of Frobenius . . . . .	187
Appendix by Walter Feit. Exceptional Subgroups of $GL_2$ . . . . .	198
<b>Part V. <math>p</math>-Adic Distributions</b> . . . . .	<b>205</b>
Chapter XII. General Distributions . . . . .	207
§ 1. Definitions . . . . .	207
§ 2. Averaging Operators . . . . .	210
§ 3. The Iwasawa Algebra . . . . .	217
§ 4. Weierstrass Preparation Theorem . . . . .	219
§ 5. Modules over $\mathbf{Z}_p[[T]]$ . . . . .	221
Chapter XIII. Bernoulli Numbers and Polynomials . . . . .	228
§ 1. Bernoulli Numbers and Polynomials . . . . .	228
§ 2. The Integral Distribution . . . . .	233
§ 3. $L$ -Functions and Bernoulli Numbers . . . . .	236
Chapter XIV. The Complex $L$ -Functions . . . . .	240
§ 1. The Hurwitz Zeta Function . . . . .	240
§ 2. Functional Equation . . . . .	244
Chapter XV. The Hecke–Eisenstein and Klein Forms . . . . .	247
§ 1. Forms of Weight 1 . . . . .	247
§ 2. The Klein Forms . . . . .	251
§ 3. Forms of Weight 2 . . . . .	252
Bibliography . . . . .	255
Subject Index . . . . .	260