

Table of contents

Chapter I. Forming geometrical intuition; statement of the main problem.	1
§1. Formulating the problem	1
§2. Spherical geometry	4
§3. Geometry on a cylinder	11
3.1. First acquaintance	11
3.2. How to measure distances	15
3.3. The study of geometry on a cylinder	20
§4. A world in which right and left are indistinguishable	24
§5. A bounded world	30
5.1. Description of the geometry	30
5.2. Lines on the torus	36
5.3. Some applications	41
§6. What does it mean to specify a geometry?	45
6.1. The definition of a geometry	45
6.2. Superposing geometries	50
Chapter II. The theory of 2-dimensional locally Euclidean geometries	53
§7. Locally Euclidean geometries and uniformly discontinuous groups of motions of the plane	53
7.1. Definition of equivalence by means of motions	53
7.2. The geometry corresponding to a uniformly discontinuous group	61
§8. Classification of all uniformly discontinuous groups of motions of the plane	66
8.1. Motions of the plane	66
8.2. Classification: generalities and groups of Type I and II	72
8.3. Classification: groups of Type III	76
§9. A new geometry	88
§10. Classification of all 2-dimensional locally Euclidean geometries	97
10.1. Constructions in an arbitrary geometry	98
10.2. Coverings	102
10.3. Construction of the covering	107
10.4. Construction of the group	113
10.5. Conclusion of the proof of Theorem 1	117

Chapter III. Generalisations and applications	121
§11. 3-dimensional locally Euclidean geometries	121
11.1. Motions of 3-space	121
11.2. Uniformly discontinuous groups in 3-space: generalities	125
11.3. Uniformly discontinuous groups in 3-space: classification	130
11.4. Orientability of the geometries	139
§12. Crystallographic groups and discrete groups	149
12.1. Symmetry groups	149
12.2. Crystals and crystallographic groups	153
12.3. Crystallographic groups and geometries: discrete groups	160
12.4. A typical example: the geometry of the rectangle	166
12.5. Classification of all locally C_n or D_n geometries	170
12.6. On the proof of Theorems 1 and 2	184
12.7. Crystals and their molecules	185
Chapter IV. Geometries on the torus, complex numbers and Lobachevsky geometry.	187
§13. Similarity of geometries	187
13.1. When are two geometries defined by uniformly discontinuous groups the same?	187
13.2. Similarity of geometries	191
§14. Geometries on the torus	196
14.1. Geometries on the torus and the modular figure	196
14.2. When do two pairs of vectors generate the same lattice?	202
14.3. Application to number theory	206
§15. The algebra of similarities: complex numbers	210
15.1. The geometrical definition of complex numbers	210
15.2. Similarity of lattices and the modular group	215
§16. Lobachevsky geometry	220
16.1. 'Motions'	220
16.2. 'Lines'	223
16.3. Distance	225
16.4. Construction of the geometry concluded	232
§17. The Lobachevsky plane, the modular group, the modular figure and geometries on the torus	238
17.1. Discreteness of the modular group	238
17.2. The set of all geometries on the torus	240
Historical remarks	245
List of notation	247
Index	249