

Contents

Chapter I. Foundations of the Theory of Algebraic Function Fields	1
I.1 Places	1
I.2 The Rational Function Field	8
I.3 Independence of Valuations	11
I.4 Divisors	14
I.5 The Riemann-Roch Theorem	22
I.6 Some Consequences of the Riemann-Roch Theorem	29
I.7 Local Components of Weil Differentials	35
Chapter II. Geometric Goppa Codes	39
II.1 Codes	39
II.2 Geometric Goppa Codes	42
II.3 Geometric Goppa Codes Associated with the Rational Function Field	49
Chapter III. Extensions of Algebraic Function Fields	58
III.1 Algebraic Extensions of Function Fields	59
III.2 Subrings of Function Fields	67
III.3 Local Integral Bases	71
III.4 The Cotrace of Weil Differentials and the Hurwitz Genus Formula	80
III.5 The Different	89
III.6 Constant Field Extensions	101
III.7 Galois Extensions I	109
III.8 Galois Extensions II	118
III.9 Inseparable Extensions	127
III.10 Estimates for the Genus of a Function Field	129
Chapter IV. Differentials of Algebraic Function Fields	134
IV.1 Derivations and Differentials	134
IV.2 The P -adic Completion	140
IV.3 Differentials and Weil Differentials	148

Chapter V. Algebraic Function Fields over Finite Constant Fields	158
V.1 The Zeta Function of a Function Field	158
V.2 The Hasse-Weil Theorem	169
V.3 Improvements of the Hasse-Weil Bound	180
Chapter VI. Examples of Algebraic Function Fields	186
VI.1 Elliptic Function Fields	186
VI.2 Hyperelliptic Function Fields	193
VI.3 Tame Cyclic Extensions of the Rational Function Field	196
VI.4 Some Elementary Abelian p -Extensions of $K(x)$, $\text{char } K = p > 0$	200
Chapter VII. More about Geometric Goppa Codes	204
VII.1 The Residue Representation of $C_{\Omega}(D, G)$	204
VII.2 Long Codes	205
VII.3 Automorphisms	209
VII.4 Hermitian Codes	211
VII.5 Decoding Geometric Goppa Codes	215
Chapter VIII. Subfield Subcodes and Trace Codes	221
VIII.1 On the Dimension of Subfield Subcodes and Trace Codes	221
VIII.2 Weights of Trace Codes	225
Appendix A. Field Theory	235
Appendix B. Algebraic Curves and Algebraic Function Fields	242
Bibliography	251
List of Notation	253
Index	257