

# CONTENTS

Introduction. ....	1
<b>Chapter 1. Gaussian diffusions</b>	
1. Gaussian diffusion. Probabilistic and analytic approaches. ....	17
2. Classification of Gaussian diffusions by the Young schemes. ....	20
3. Long time behaviour of the Green functions of Gaussian diffusions. ....	25
4. Complex stochastic Gaussian diffusion. ....	28
5. The rate of escape for Gaussian diffusions and scattering for its perturbations. ....	34
<b>Chapter 2. Boundary value problem for Hamiltonian systems</b>	
1. Rapid course in calculus of variations. ....	40
2. Boundary value problem for non-degenerate Hamiltonians. ....	50
3. Regular degenerate Hamiltonians of the first rank. ....	59
4. General regular Hamiltonians depending quadratically on momenta. ....	72
5. Hamiltonians of exponential growth in momenta. ....	75
6. Complex Hamiltonians and calculus of variations of saddle-points. ....	87
7. Stochastic Hamiltonians. ....	92
<b>Chapter 3. Semiclassical approximation for regular diffusion</b>	
1. Main ideas of the WKB-method with imaginary phase. ....	97
2. Calculation of the two-point function for regular Hamiltonians. ....	104
3. Asymptotic solutions of the transport equation. ....	110
4. Local asymptotics of the Green function for regular Hamiltonians. ....	112
5. Global small diffusion asymptotics and large deviations. ....	119
6. Asymptotics for non-regular diffusion: an example. ....	124
7. Analytic solutions to some linear PDE. ....	128
<b>Chapter 4. Invariant degenerate diffusion on cotangent bundles</b>	
1. Curvilinear Ornstein-Uhlenbeck process and stochastic geodesic flow. ..	136
2. Small time asymptotics for stochastic geodesic flow. ....	140
3. The trace of the Green function and geometric invariants. ....	143
<b>Chapter 5. Transition probability densities for stable jump-diffusion</b>	
1. Asymptotic properties of one-dimensional stable laws. ....	146
2. Asymptotic properties of finite-dimensional stable laws. ....	149
3. Transition probability densities for stable jump-diffusion. ....	161
4. Stable jump-diffusions combined with compound Poisson processes. ....	178
5. Stable-like processes. ....	182
6. Applications to the sample path properties of stable jump-diffusions. ...	187

## Chapter 6. Semiclassical asymptotics for the localised Feller-Courrège processes

1. Maslov's tunnel equations and the Feller-Courrège processes. ....	191
2. Rough local asymptotics and local large deviations. ....	194
3. Refinement and globalisation. ....	217

## Chapter 7. Complex stochastic diffusions or stochastic Schrödinger equations

1. Semiclassical approximation: formal asymptotics. ....	223
2. Semiclassical approximation: justification and globalisation. ....	229
3. Applications: two-sided estimates to complex heat kernels, large deviation principle, well-posedness of the Cauchy problem. ....	235
4. Path integration and infinite-dimensional saddle-point method. ....	236

## Chapter 8. Some topics in semiclassical spectral analysis

1. Double-well splitting. ....	239
2. Low lying eigenvalues of diffusion operators and the life-times of diffusion processes. ....	247
3. Quasi-modes of diffusion operators around a closed orbit of the corresponding classical system. ....	252

## Chapter 9. Path integration for the Schrödinger, heat and complex stochastic diffusion equations

1. Introduction. ....	255
2. Path integral for the Schrödinger equation in $p$ -representation. ....	263
3. Path integral for the Schrödinger equation in $x$ -representation. ....	267
4. Singular potentials. ....	269
5. Semiclassical asymptotics. ....	272
6. Fock space representation. ....	276

## Appendices

A. Main equation of the theory of continuous quantum measurements. ....	280
B. Asymptotics of Laplace integrals with complex phase. ....	283
C. Characteristic functions of stable laws. ....	293
D. Lévy-Khintchine $\Psi DO$ and Feller-Courrège processes. ....	298
E. Equivalence of convex functions. ....	303
F. Unimodality of symmetric stable laws. ....	305
G. Infinite divisible complex distributions and complex Markov processes. ....	312
H. A review of main approaches to the rigorous construction of path integral. ....	322
I. Perspectives and problems. ....	326

References .....	329
------------------	-----

Main notations .....	346
----------------------	-----

Subject Index .....	347
---------------------	-----