Contents

1 Mathematical fundamentals 1

1.1 Simple tensors 1
1.2 General tensors 3
1.3 Special tensors 7
1.4 Orthogonal tensors 15
1.5 Spherical tensor, deviatoric tensor 16
1.6 Differential operators 17
1.7 Differentiation rules 19
1.8 Invariants of a second-order tensor 22
1.9 The eigenvalue problem of a second-order tensor 23
1.10 Rotation tensor, rotation vector 29
1.11 Analytical solution of eigenvalue-problems 33
1.12 Tensor functions on the basis of power series 35
1.13 Exponential, skew-symmetric tensors 36
1.14 Summary of notations and formulae 38
Exercises 41

2 Deformation 43

2.1 General backgrounds 43
2.2 Deformation gradient 47
2.3 Deformation gradient in material and spatial coordinates 54
2.4 Polar decomposition 58
2.5 Green-Lagrange strain tensor, Almansi strain tensor 66
2.6 Eigenvectors and eigenvalues of deformation variable 71
2.7 Unified definitions of strain tensors 78
2.8 Isochoric and volumetric deformations 81
2.9 Rate of deformation tensor and spin tensor 82
2.10 Pull-back and push-forward operations 88
2.11 Isotropic tensor functions of $\mathbf{C}$ and $\mathbf{b}$ 92
Exercises 97

3 Stresses 99

3.1 Cauchy Stress tensor 99
3.2 Stress tensors 104
3.3 Energy conjugate stress and strain variables 109
3.4 Summary of important definitions 112
Exercises 113

4 Time derivative 115

4.1 Definitions 115
4.2 Velocity and acceleration 117
4.3 Examples for material time derivative 118
Exercises 122

5 Balance laws 123

5.1 Conservation of mass 123
5.2 Balance of momentum 124
5.3 Balance of moment of momentum 127
5.4 Balance of kinetic energy 129
5.5 Conservation of energy 131
5.6 Principle of virtual work 134
Exercises 137

6 Constitutive modelling 139

6.1 General principles 139
6.2 Objective tensors 142
6.3 Elastic material 144
6.4 Isotropic elastic material 146
6.5 Derivatives of scalar-valued functions 148
6.6 Hyperelastic material or Green-elastic materials 152
6.7 Isotropic hyperelastic material 156
6.8 Special constitutive models for isotropic hyperelasticity 159
6.9 ST.VENANT-KIRCHHOFF material 163
6.10 HOOKEAN material 166
6.11 Linearization and comparison of various material models 170
Exercises 174

Appendix 1 175

A1.1 Index notation 175
A1.2 Metric tensor and geometrical properties 176
A1.3 Vector decompositions, tensor components of first order 179
A1.4 Definition of higher-order tensor components 179
A1.5 Permutation tensor 181
A1.6 Christoffel symbols, covariant differentiation 182

References 185

Index 191