

# Contents

<b>Preface</b> . . . . .	xi
--------------------------	----

## Part A: Ordering the Braids

<b>I. Braids vs. Self-Distributive Systems</b> . . . . .	1
I.1 Braid Groups . . . . .	4
I.2 Braid Colourings . . . . .	13
I.3 A Self-Distributive Operation on Braids . . . . .	26
I.4 Extended Braids . . . . .	35
I.5 Notes . . . . .	44
<b>II. Word Reversing</b> . . . . .	47
II.1 Complemented Presentations . . . . .	48
II.2 Coherent Complements . . . . .	58
II.3 Garside Elements . . . . .	68
II.4 The Case of Braids . . . . .	78
II.5 Double Reversing . . . . .	87
II.6 Notes . . . . .	94
<b>III. The Braid Order</b> . . . . .	97
III.1 More about Braid Colourings . . . . .	98
III.2 The Linear Ordering of Braids . . . . .	106
III.3 Handle Reduction . . . . .	115
III.4 Alternative Definitions . . . . .	127
III.5 Notes . . . . .	137

<b>IV. The Order on Positive Braids</b> . . . . .	141
IV.1 The Case of Three Strands . . . . .	142
IV.2 The General Case . . . . .	150
IV.3 Applications . . . . .	165
IV.4 Notes . . . . .	170
IV.Appendix: Rank Tables . . . . .	172

## Part B: Free LD-systems

<b>V. Orders on Free LD-systems</b> . . . . .	175
V.1 Free Systems . . . . .	178
V.2 The Absorption Property . . . . .	186
V.3 The Confluence Property . . . . .	193
V.4 The Comparison Property . . . . .	200
V.5 Canonical Orders . . . . .	209
V.6 Applications . . . . .	216
V.7 Notes . . . . .	231
<b>VI. Normal Forms</b> . . . . .	235
VI.1 Terms as Trees . . . . .	236
VI.2 The Cuts of a Term . . . . .	242
VI.3 The $\partial$ -Normal Form . . . . .	250
VI.4 The Right Normal Form . . . . .	256
VI.5 Applications . . . . .	271
VI.6 Notes . . . . .	276
VI.Appendix: The First Codes . . . . .	280
<b>VII. The Geometry Monoid</b> . . . . .	285
VII.1 Left Self-Distributivity Operators . . . . .	286
VII.2 Relations in the Geometry Monoid . . . . .	301
VII.3 Syntactic Proofs . . . . .	310
VII.4 Cuts and Collapsing . . . . .	322
VII.5 Notes . . . . .	329

<b>VIII. The Group of Left Self-Distributivity</b>	331
VIII.1 The Group $G_{LD}$ and the Monoid $M_{LD}$	332
VIII.2 The Blueprint of a Term	343
VIII.3 Order Properties in $G_{LD}$	356
VIII.4 Parabolic Subgroups	367
VIII.5 Simple Elements in $M_{LD}$	371
VIII.6 Notes	383
<b>IX. Progressive Expansions</b>	385
IX.1 The Polish Algorithm	386
IX.2 The Content of a Term	395
IX.3 Perfect Terms	401
IX.4 Convergence Results	415
IX.5 Effective Decompositions	421
IX.6 The Embedding Conjecture	428
IX.7 Notes	436
IX.Appendix: Other Algebraic Identities	439

## Part C: Other LD-Systems

<b>X. More LD-Systems</b>	443
X.1 The Laver Tables	446
X.2 Finite Monogenic LD-systems	457
X.3 Multi-LD-Systems	467
X.4 Idempotent LD-systems	472
X.5 Two-Sided Self-Distributivity	476
X.6 Notes	480
X.Appendix: The First Laver Tables	485
<b>XI. LD-Monoids</b>	489
XI.1 LD-Monoids	490
XI.2 Completion of an LD-System	498
XI.3 Free LD-Monoids	511
XI.4 Orders on Free LD-Monoids	517
XI.5 Extended Braids	527
XI.6 Notes	534

<b>XII. Elementary Embeddings</b> . . . . .	537
XII.1 Large Cardinals . . . . .	538
XII.2 Elementary Embeddings . . . . .	546
XII.3 Operations on Elementary Embeddings . . . . .	553
XII.4 Finite Quotients . . . . .	559
XII.5 Notes . . . . .	568
<b>XIII. More about the Laver Tables</b> . . . . .	571
XIII.1 A Dictionary . . . . .	572
XIII.2 Computation of Rows . . . . .	577
XIII.3 Fast Growing Functions . . . . .	583
XIII.4 Lower Bounds . . . . .	591
XIII.5 Notes . . . . .	598
<b>Bibliography</b> . . . . .	603
<b>List of Symbols</b> . . . . .	615
<b>Index</b> . . . . .	619