

Contents

8. The biharmonic equation	1
8.1 The concept of a continuum	3
8.2 Displacements and strains in continua	5
8.2.1 Physical interpretations of the strain matrix	12
8.2.2 Physical interpretation of the rotation matrix	16
8.2.3 Indicinal notation representations of strain and rotation	18
8.2.4 Transformation of the strain matrix	19
8.2.5 Principal strains and strain invariants	24
8.2.6 Compatibility of strains	26
8.3 Stresses in a continuum	36
8.3.1 The stress dyadic and the stress matrix	37
8.3.2 Tractions on an arbitrary plane	39
8.3.3 Equations of equilibrium	41
8.3.4 Symmetry of the stress matrix	43
8.3.5 Sign convention for stresses	45
8.3.6 Transformation of the stress matrix	47
8.3.7 Principal stresses and stress invariants	49
8.4 Constitutive equations for linear elastic solids	57
8.4.1 Generalized Hooke's Law	57
8.4.2 The strain energy density	58
8.4.3 Symmetry of the elasticity matrix	60
8.4.4 Isotropic elastic material	62
8.4.5 Thermodynamic constraints on the elastic constants	66
8.4.6 Boundary conditions	68
8.4.7 Time derivatives	71
8.5 Uniqueness theorem in the classical theory of elasticity	72
8.6 Plane problems in classical elasticity	81
8.7 The Airy stress function	92
8.8 Methods of solution of the biharmonic equation	104

8.8.1	Series solution in Cartesian coordinates	105
8.8.2	Variables separable solution - rectangular Cartesian coordinates	118
8.8.3	Integral transform solution of the biharmonic equation governing plane problems	130
8.8.4	Line load problems for an infinite plane	138
8.8.5	Application of complex variable methods	145
8.9	Polar coordinate formulation	
	of the plane problem in elasticity	151
8.9.1	Definition of stresses in plane polar coordinates	152
8.9.2	Definition of strains in plane polar coordinates	154
8.9.3	Elastic stress-strain relations in polar coordinates	158
8.9.4	Transformation from Cartesian equations	159
8.9.5	The Airy stress function in plane polar coordinates	161
8.9.6	Boundary conditions in plane polar coordinates	162
8.9.7	Development of the Airy stress function in polar coordinates	164
8.9.8	An application of complex variable techniques	176
8.10	Biharmonic function formulation	
	of three-dimensional problems in elasticity	181
8.10.1	The strain potential	182
8.10.2	The Galerkin vector	184
8.10.3	General properties of biharmonic functions	186
8.10.4	Love's strain function	193
8.10.5	Axisymmetric problem formulation	197
8.10.6	Polynomial solutions of the biharmonic equation; the interior solution	199
8.10.7	Love's strain function approach: spherical polar coordinate formulation for unbounded domains	209
8.10.8	Kelvin's problem	213
8.10.9	A centre of dilatation	222
8.10.10	Boussinesq's problem	225
8.10.11	The spherical cavity problem	233
8.10.12	Application of integral transforms	238
8.10.13	Mixed boundary value problems	258
8.11	Flexure of thin elastic plates	267
8.11.1	Deformation of the plate region	269
8.11.2	Flexural stresses and stress resultants	279
8.11.3	Plate stress-strain relations	282
8.11.4	Equation of equilibrium for the plate	286
8.11.5	Strain energy of a plate	289

8.11.6	The principle of virtual work	291
8.11.7	Boundary conditions for plate problems	293
8.11.8	The classical theory of thin plates	297
8.11.9	Flexure of thin circular plates	305
8.11.10	Complex variable method for circular plates	325
8.11.11	Green's function for a thin plate	330
8.11.12	Flexure of rectangular plates	332
8.11.13	Certain general solutions of the biharmonic equation	339
8.11.14	Navier and Levy solutions for rectangular plates	345
8.11.15	Application of integral transform techniques	366
8.11.16	Complex variable methods for the solution of plate problems	374
8.11.17	Uniqueness of solution governing deflections of a plate	386
8.11.18	Uniqueness of solution for plates with clamped boundaries	390
8.12	Slow viscous flow	393
8.12.1	Kinematics of fluid flow	394
8.12.2	Equation of motion	397
8.12.3	Constitutive equations for a viscous fluid	397
8.12.4	Thermodynamic constraints on the viscosity coefficients	399
8.12.5	Navier-Stokes equation	403
8.12.6	Biharmonic formulations of problems in slow viscous flow	404
8.12.7	Planar problems in slow viscous flow	407
8.12.8	Axisymmetric problems in slow viscous flow	409
8.12.9	Boundary conditions	411
8.12.10	Stokes' paradox and related problems	419
8.12.11	Slow viscous flow past a sphere	424
8.12.12	Application of integral transform techniques	430
8.12.13	Diffusive motions in viscous fluids	440
8.13	The compatibility conditions	454
8.14	A proof of Stokes' paradox	459
8.15	A uniqueness theorem for viscous flows	462
8.16	PROBLEM SET 8	465
9.	Poisson's equation	503
9.1	Flow through porous media with internal sources	504
9.2	Heat conduction in the presence of heat sources	507

9.3	Transverse deflections of a stretched transversely loaded membrane	508
9.4	Boundary conditions	508
9.4.1	Boundary conditions for flow in porous media	509
9.4.2	Boundary conditions for heat conduction	510
9.4.3	Boundary conditions for loaded membranes	511
9.5	Generalized results	513
9.6	Green's function for Poisson's equation	526
9.6.1	Integral transform techniques	531
9.6.2	Series solution techniques	535
9.6.3	Method of eigenfunctions	540
9.6.4	Symmetry of the Green's function	551
9.6.5	The method of images	553
9.7	A monotonicity result for Poisson's equation	557
9.8	Viscous flow in conduits	559
9.9	Torsion of prismatic elastic solids	565
9.9.1	General formulation of the torsion problem	568
9.9.2	Torsion of prismatic elements with multiply connected cross-sections	577
9.9.3	Solutions for torsion problems derived from equations of boundaries	581
9.9.4	Variables separable solutions for the torsion problem	587
9.9.5	A complex variable formulation of the torsion problem	600
9.10	An alternative derivation of the elastic torsion problem	608
9.11	The gravitational potential	614
9.11.1	Spatial distribution of matter	620
9.12	PROBLEM SET 9	634
	Bibliography	649
	Index	679