

# Table of Contents

Foreword .....	v
Editorial Remarks .....	vii
Table of Contents .....	ix
List of Contributing Authors .....	xv
1 Development, Characterization, Prospects .....	1
1.1 Historical Remarks .....	1
1.2 General Characterization .....	1
1.3 Impact on Education .....	2
1.4 Impact on Research .....	4
1.5 Computer Algebra – Today and Tomorrow .....	6
1.5.1 Today .....	6
1.5.2 Outlook .....	7
2 Topics of Computer Algebra .....	11
2.1 Exact Arithmetic .....	11
2.1.1 Long Integer Arithmetic .....	11
2.1.2 Arithmetic with Polynomials, Rational Functions and Power Series .....	13
2.1.3 Euclid’s Algorithm and Continued Fractions .....	16
2.1.4 Modular Arithmetic and the Chinese Remainder Theorem ..	17
2.1.5 Computations with Algebraic Numbers .....	18
2.1.6 Real Algebraic Numbers .....	19
2.1.7 $p$ -adic Numbers and Approximations .....	20
2.1.8 Finite Fields .....	21
2.2 Algorithms for Polynomials and Power Series .....	23
2.2.1 The Division Algorithm .....	23
2.2.2 Factorization of Polynomials .....	24
2.2.3 Absolute Factorization of Polynomials .....	26
2.2.4 Polynomial Decomposition .....	26
2.2.5 Gröbner Bases .....	28
2.2.6 Standard Bases .....	32
2.2.7 Characteristic Sets .....	32
2.2.8 Algorithmic Invariant Theory .....	33
2.3 Linear Algebra .....	36
2.3.1 Linear Systems .....	36
2.3.2 Algorithms for Matrix Canonical Forms .....	38
2.4 Constructive Methods of Number Theory .....	41
2.4.1 Primality Tests .....	41
2.4.2 Integer Factorization .....	44
2.4.3 Algebraic Number Fields and Algebraic Function Fields ...	45
2.4.4 Galois Groups .....	47
2.4.5 Rational Points on Elliptic Curves .....	48
2.4.6 Geometry of Numbers .....	50
2.5 Algorithms of Commutative Algebra and Algebraic Geometry ...	51
2.5.1 Algorithms for Polynomial Ideals and Their Varieties .....	51

2.5.2	Singularities of Varieties . . . . .	54
2.5.3	Real Algebraic Geometry . . . . .	55
2.6	Algorithmic Aspects of the Theory of Algebras . . . . .	57
2.6.1	Structure Constants . . . . .	58
2.6.2	Generators and Relations, Swapping and G-Algebras . . . . .	58
2.6.3	Monad Algebras, Path Algebras and Generalizations . . . . .	59
2.6.4	Finite-Dimensional Lie Algebras . . . . .	60
2.6.5	Non-commutative Gröbner Bases . . . . .	60
2.6.6	Structural Issues and Classification . . . . .	63
2.6.7	Identities . . . . .	63
2.6.8	Computational Aspects in the Representation Theory of Quivers and Path Algebras . . . . .	64
2.7	Computational Group Theory . . . . .	65
2.7.1	A Crash Course in Group Theory . . . . .	66
2.7.2	Describing Groups . . . . .	67
2.7.3	A Brief History . . . . .	69
2.7.4	Permutation Groups . . . . .	71
2.7.5	Matrix Groups . . . . .	74
2.7.6	Black Box Groups . . . . .	75
2.7.7	Abelian Groups . . . . .	76
2.7.8	Polycyclic Groups . . . . .	76
2.7.9	Finitely Presented Groups . . . . .	78
2.7.10	Group-Theoretic Software . . . . .	83
2.7.11	Another Perspective . . . . .	83
2.8	Algorithms of Representation Theory . . . . .	84
2.8.1	Ordinary Representation Theory . . . . .	84
2.8.2	Modular Representation Theory . . . . .	85
2.8.3	Generic Character Tables . . . . .	87
2.8.4	Summary of Systems . . . . .	88
2.9	Algebraic Methods for Constructing Discrete Structures . . . . .	89
2.10	Summation and Integration . . . . .	91
2.10.1	Definite Summation and Hypergeometric Identities . . . . .	91
2.10.2	Symbolic Integration . . . . .	94
2.11	Symbolic Methods for Differential Equations . . . . .	96
2.11.1	Introduction . . . . .	96
2.11.2	Differential Galois Theory . . . . .	97
2.11.3	Lie Symmetries . . . . .	98
2.11.4	Painlevé Theory . . . . .	99
2.11.5	Completion . . . . .	102
2.11.6	Differential Ideal Theory . . . . .	104
2.11.7	Dynamical Systems . . . . .	105
2.11.8	Numerical Analysis . . . . .	108
2.12	Symbolic/Numeric Methods . . . . .	109
2.12.1	Computer Analysis . . . . .	109
2.12.2	Algorithms for Computing Validated Results . . . . .	110

2.12.3	Hybrid Methods . . . . .	112
2.13	Algebraic Complexity Theory . . . . .	125
2.14	Coding Theory and Cryptography . . . . .	128
2.14.1	Coding Theory . . . . .	128
2.14.2	Quantum Coding Theory . . . . .	130
2.14.3	Cryptography . . . . .	131
2.15	Algorithmic Methods in Universal Algebra and Logic . . . . .	132
2.15.1	Term Rewriting Systems . . . . .	132
2.15.2	Decision Procedures and Quantifier Elimination Meth- ods for Algebraic Theories . . . . .	137
2.16	Knowledge Representation and Abstract Data Types . . . . .	140
2.16.1	Mathematical Knowledge Representation and Expert Systems . . . . .	140
2.16.2	Abstract Data Types . . . . .	142
2.17	On the Design of Computer Algebra Systems . . . . .	143
2.17.1	Memory Management . . . . .	143
2.17.2	Program Verification and Abstract Data Types . . . . .	144
2.17.3	The Concept of Types . . . . .	144
2.17.4	Genericity . . . . .	145
2.17.5	Modularization . . . . .	145
2.17.6	Parallel Implementation . . . . .	145
2.17.7	Continuing Development of Computer Algebra Systems . . . . .	146
2.18	Parallel Computer Algebra Systems . . . . .	146
2.18.1	Parallel Architectures and Operating Systems Supports . . . . .	146
2.18.2	Parallel Execution: Mapping and Scheduling . . . . .	147
2.18.3	Parallelism Expression and Languages . . . . .	149
2.19	Interfaces and Standardization . . . . .	150
2.19.1	Interfaces to Word Processors . . . . .	150
2.19.2	Graphics . . . . .	150
2.19.3	Interfaces to Numerical Software . . . . .	150
2.19.4	User Interfaces . . . . .	152
2.19.5	General Problem-Solving Environments . . . . .	152
2.19.6	Standardisation . . . . .	153
2.19.7	MathML . . . . .	154
2.20	Hardware Implementation of Computer Algebra Algorithms . . . . .	161
3	Applications of Computer Algebra . . . . .	163
3.1	Physics . . . . .	163
3.1.1	Elementary Particle Physics . . . . .	164
3.1.2	Gravity . . . . .	172
3.1.3	'Central Configurations' in the Newtonian N-Body Problem of Celestial Mechanics . . . . .	176
3.1.4	CA-Systems for Differential Geometry and Applications . . . . .	180
3.1.5	Differential Equations in Physics . . . . .	187
3.2	Mathematics . . . . .	195
3.2.1	Computer Algebra in Group Theory . . . . .	196

3.2.2	The Tangent Cone Algorithm and Applications in the Theory of Singularities .....	197
3.2.3	Automatic Theorem Proving in Geometry .....	201
3.2.4	Homological Algebra .....	207
3.2.5	Study of Differential Structures on Quantum Groups .....	212
3.2.6	Orthogonal Polynomials and Computer Algebra .....	214
3.2.7	Computer Algebra in Symmetric Bifurcation Theory .....	215
3.2.8	Symbolic-Numeric Treatment of Equivariant Systems of Equations .....	216
3.3	Computer Science .....	217
3.3.1	Computer Algebra in Computer Science .....	217
3.3.2	Decomposable Structures, Generating Functions and Average-Case of Algorithms .....	219
3.3.3	Telecommunication Management Networks .....	221
3.4	Engineering .....	221
3.4.1	Computer Algebra, a Modern Research Tool for Engineering .....	221
3.4.2	Critical Load Computations for Jet Engines .....	226
3.4.3	Audio Signal Processing .....	227
3.4.4	Robotics .....	229
3.4.5	Computer Aided Design and Modelling .....	234
3.5	Chemistry .....	242
3.5.1	Computer Algebra in Chemistry and Crystallography .....	242
3.5.2	Chemical Reaction Systems .....	243
3.6	Computer Algebra in Education .....	244
3.6.1	New Hand-Held Computer Symbolic Algebra Tools in Mathematics Education .....	245
3.6.2	The Dutch Perspective .....	247
3.6.3	Computer Algebra in Teaching and Learning Mathematics: Experiences at the University of Plymouth, England .....	250
3.6.4	The Educational Use of Computer Algebra Systems at the University of Illinois .....	253
3.6.5	Mathematics Education from a MATHEMATICA Perspective .....	254
3.6.6	Visualization: Courseware for Mathematics Education .....	256
4	Computer Algebra Systems .....	261
4.1	General Purpose Systems .....	261
4.1.1	AXIOM .....	261
4.1.2	Aldor .....	265
4.1.3	DERIVE and the TI-92 .....	271
4.1.4	Macsyma .....	283
4.1.5	MAGMA .....	295
4.1.6	Maple .....	308
4.1.7	<i>Mathematica</i> .....	314

4.1.8	<i>MuPAD</i> .....	321
4.1.9	REDUCE .....	333
4.2	Special Purpose Systems .....	345
4.2.1	Algebraic Combinatorics Environment (ACE) .....	345
4.2.2	Building Nonassociative Algebras With Albert .....	346
4.2.3	ALGEB .....	348
4.2.4	AMORE .....	348
4.2.5	BERGMAN .....	349
4.2.6	CANNES / PARCAN .....	351
4.2.7	CARAT .....	354
4.2.8	CASA .....	356
4.2.9	CHEVIE .....	359
4.2.10	C-Meataxe .....	363
4.2.11	CoCoA .....	364
4.2.12	CREP .....	368
4.2.13	The Desir Project and Its Continuation .....	370
4.2.14	DISCRETA: A Tool for Constructing $t$ -Designs .....	372
4.2.15	FELIX .....	375
4.2.16	<i>Fermat</i> .....	380
4.2.17	FoxBox and Other Blackbox Systems .....	383
4.2.18	GAP .....	385
4.2.19	GiNaC .....	391
4.2.20	Kan/sm1 .....	392
4.2.21	KANT V4 .....	396
4.2.22	LiDIA .....	403
4.2.23	Lie .....	408
4.2.24	LIE .....	411
4.2.25	A Brief Introduction to Macaulay 2 .....	411
4.2.26	MAS .....	421
4.2.27	MASYCA .....	428
4.2.28	MOC .....	429
4.2.29	NTL: A Library for Doing Number Theory .....	430
4.2.30	PARI .....	431
4.2.31	PARSAC .....	434
4.2.32	QUOTPIC .....	436
4.2.33	REDUX .....	437
4.2.34	REPTILES A Program for Interactively Generating Periodic Tilings .....	438
4.2.35	SAC-1, Aldes/SAC-2, Saclib .....	439
4.2.36	SciNapse: Software that Writes PDE Software .....	440
4.2.37	SENAC .....	441
4.2.38	SIMATH - Algorithms in Number Theory .....	442
4.2.39	SINGULAR - A Computer Algebra System for Poly- nomial Computations .....	445
4.2.40	SymbMath .....	451

4.2.41	SYMMETRICA . . . . .	452
4.2.42	Theorema: Computation and Deduction in Natural Style . .	453
4.2.43	THEORIST—a User Interface for Symbolic Algebra . . . . .	454
4.3	Packages . . . . .	459
4.3.1	ANU Polycyclic Quotient Programs . . . . .	459
4.3.2	AREP . . . . .	461
4.3.3	CALI . . . . .	463
4.3.4	CLN . . . . .	464
4.3.5	CRACK, LIEPDE, APPLYSYM and CONLAW . . . . .	465
4.3.6	DIMSYM . . . . .	468
4.3.7	EinS . . . . .	469
4.3.8	<i>FeynArts</i> and <i>FormCalc</i> . . . . .	469
4.3.9	FeynCalc – Tools and Tables for Elementary Particle Physics . . . . .	471
4.3.10	GRAPE . . . . .	473
4.3.11	Recognising Matrix Groups over Finite Fields . . . . .	474
4.3.12	MOLGEN . . . . .	476
4.3.13	ORME . . . . .	477
4.3.14	Ratappr . . . . .	477
4.3.15	TTC: Tools of Tensor Calculus . . . . .	480
5	Meetings and Publications . . . . .	485
5.1	Conferences and Proceedings . . . . .	485
5.2	Books on Computer Algebra . . . . .	490
	Cited References . . . . .	493
	Subject Index . . . . .	623
	Index for Authors' Contributions . . . . .	635