

# Contents

<b>Preface</b> . . . . .	v
<b>A Word on Notation</b> . . . . .	ix
<b>Introduction</b> . . . . .	1
<b>1. The First Main Theorem</b> . . . . .	5
§1.1. The Poisson-Jensen Formula . . . . .	5
§1.2. The Nevanlinna Functions . . . . .	12
Counting Functions . . . . .	12
Mean Proximity Functions . . . . .	13
Height or Characteristic Functions . . . . .	17
§1.3. The First Main Theorem . . . . .	17
§1.4. Ramification and Wronskians . . . . .	20
§1.5. Nevanlinna Functions for Sums and Products . . . . .	23
§1.6. Nevanlinna Functions for Some Elementary Functions . . . . .	24
§1.7. Growth Order and Maximum Modulus . . . . .	27
§1.8. A Number Theoretic Digression: The Product Formula . . . . .	28
§1.9. Some Differential Operators on the Plane . . . . .	31
§1.10. Theorems of Stokes, Fubini, and Green-Jensen . . . . .	33
§1.11. The Geometric Interpretation of the Ahlfors-Shimizu Characteristic . . . . .	35
§1.12. Why $N$ and $\hat{T}$ are Used in Nevanlinna Theory Instead of $n$ and $A$ . . . . .	41
§1.13. Relationships Among the Nevanlinna Functions on Average . . . . .	43
§1.14. Jensen's Inequality . . . . .	47
<b>2. The Second Main Theorem via Negative Curvature</b> . . . . .	49
§2.1. Khinchin Functions and Exceptional Sets . . . . .	49
§2.2. The Nevanlinna Growth Lemma and the Height Transform . . . . .	51
§2.3. Definitions and Notation . . . . .	56
§2.4. The Ramification Theorem . . . . .	57
§2.5. The Second Main Theorem . . . . .	60
§2.6. A Simpler Error Term . . . . .	71
§2.7. The Unintegrated Second Main Theorem . . . . .	73

§2.8. A Uniform Second Main Theorem . . . . .	74
§2.9. The Spherical Isoperimetric Inequality . . . . .	78
<b>3. Logarithmic Derivatives . . . . .</b>	<b>89</b>
§3.1. Inequalities of Smirnov and Kolokolnikov . . . . .	89
§3.2. The Gol'dberg-Grinshtein Estimate . . . . .	92
§3.3. The Borel-Nevanlinna Growth Lemma . . . . .	98
§3.4. The Logarithmic Derivative Lemma . . . . .	104
§3.5. Functions of Finite Order . . . . .	106
<b>4. The Second Main Theorem via Logarithmic Derivatives . . . . .</b>	<b>107</b>
§4.1. Definitions and Notation . . . . .	107
§4.2. The Second Main Theorem . . . . .	108
§4.3. Functions of Finite Order . . . . .	117
<b>5. Some Applications . . . . .</b>	<b>119</b>
§5.1. Infinite Products . . . . .	120
§5.2. Defect Relations . . . . .	123
§5.3. Picard's Theorem . . . . .	128
§5.4. Totally Ramified Values . . . . .	129
§5.5. Meromorphic Solutions to Differential Equations . . . . .	129
§5.6. Functions Sharing Values . . . . .	135
§5.7. Bounding Radii of Discs . . . . .	136
§5.8. Theorems of Landau and Schottky Type . . . . .	141
§5.9. Slowly Moving Targets . . . . .	144
§5.10. Fixed Points and Iteration . . . . .	146
<b>6. A Further Digression into Number Theory: Theorems of Roth and Khinchin . . . . .</b>	<b>151</b>
§6.1. Roth's Theorem and Vojta's Dictionary . . . . .	151
§6.2. The Khinchin Convergence Condition . . . . .	158
<b>7. More on the Error Term . . . . .</b>	<b>161</b>
§7.1. Sharpness of the Second Main Theorem and the Logarithmic Derivative Lemma . . . . .	161
§7.2. Better Error Terms for Functions with Controlled Growth . . . . .	170
§7.3. Error Terms for Some Classical Special Functions . . . . .	177
<b>Bibliography . . . . .</b>	<b>187</b>
<b>Glossary of Notation . . . . .</b>	<b>193</b>
<b>Index . . . . .</b>	<b>197</b>