

Table of Contents

1. Elements of Fluid Mechanics	1
1.1 Fundamental Equations	2
1.1.1 Equation of Motion	2
1.1.2 Continuity Equation	2
1.1.3 Energy Equation	2
1.1.4 Equation of State	2
1.2 Solutions to the Fundamental Equations	3
1.3 Propagation of Discontinuities	4
1.3.1 Sound Speed	4
1.3.2 Speed of Discontinuity Propagation	5
1.3.3 Characteristics	6
1.3.4 Shock Waves	8
1.4 Derivation of the Hugoniot Relations	9
1.4.1 Conservation of Mass	9
1.4.2 Conservation of Momentum	10
1.4.3 Conservation of Energy	10
1.5 Rayleigh Line	11
1.6 Applications of Hugoniot Equations to a Perfect Gas	13
1.6.1 Calculation of Shock Speed	13
1.6.2 Calculation of Shock Pressure	14
1.6.3 Calculation of Volume Behind the Shock	14
1.6.4 Graphical Representation	15
1.6.5 Reflection of a Uniform Shock	15
1.6.6 Conditions Behind the First Reflected Shock from a Fixed Boundary	16
1.7 Detonation Waves	17
1.8 Elastic–Plastic Waves	17
1.9 Units and Orders of Magnitude	20
1.10 Measurements to Obtain Equation of State Data	20
1.10.1 Experimental Methods	20
1.10.2 Relation of the Free Surface Velocity to the Shock Particle Velocity in a Solid	22
1.10.3 Form of the Equation of State for Solids	23
1.10.4 Detonation Pressure Measurement	25

VIII Table of Contents

2. Numerical Techniques	27
2.1 Von Neumann Finite Difference Scheme	27
2.1.1 Time Centering	28
2.1.2 Space Centering	28
2.2 Artificial Viscosity	28
2.2.1 Generalized Artificial Viscosity	28
2.2.2 Applications of the Generalized Artificial Viscosity in One Space Dimension	29
2.3 Stability Conditions	32
2.3.1 Courant Condition	32
2.3.2 Von Neumann Stability Analysis	32
2.4 Finite Difference Scheme in Two Dimensions	33
2.4.1 Integral Definition of a Derivative	33
2.4.2 Integration Paths	34
2.4.3 Properties of the Integration Scheme	34
2.4.4 Continuity Equation	34
2.5 Finite Difference Scheme in Three Dimensions	35
2.6 Finite Difference Scheme for Double Operators in Two Dimensions	35
2.7 Grid Stabilization	36
3. Modeling the Behavior of Materials	37
3.1 Introduction	37
3.1.1 Hooke's Law	37
3.1.2 Rigid Body Rotation	39
3.2 Plastic Flow Region	39
3.2.1 Yield Strength	41
3.2.2 Von Mises Yield Condition	43
3.2.3 Plastic Strain	45
3.2.4 Tresca Yield Condition	46
3.3 Flow Stress	48
3.3.1 Strain Hardening	50
3.3.2 A General Form of Strain Hardening	50
3.4 Rate Dependent Yield Models	52
3.4.1 Maxwell Solid	52
3.4.2 Dislocation Theory	53
3.4.3 Flow Stress Measurements	57
3.5 Upper Yield Point	59
3.6 Nonhomogeneous Properties	60
3.7 Hydrostatic Pressure Equation of State	60
3.8 Modeling Fracture	62
3.8.1 Fracture Toughness Testing	65
3.8.2 Spallation	67
3.8.3 Ductile Fracture	68
3.8.4 Strain Damage	68

3.8.5	Damage in Elastic Regime	69
3.8.6	Computer Simulation of Fracture	70
3.8.7	Damage in Plastic Regime	71
3.9	Equation of State of Explosive Detonation Products	75
3.9.1	Numerical Calculation of a Detonation	79
4.	Two-Dimensional Elastic–Plastic Flow	83
4.1	Fundamental Equations	83
4.1.1	Equation of Motion in x, y Coordinates with Cylindrical Symmetry and Rotation About the x Axis	83
4.1.2	Conservation of Mass	84
4.1.3	First Law of Thermodynamics	84
4.1.4	Velocity Strains	84
4.1.5	Stress Deviator Tensor	85
4.1.6	Pressure Equation of State	85
4.1.7	Total Stresses	85
4.1.8	Artificial Viscosity	85
4.1.9	Von Mises Yield Condition	86
4.2	Finite Difference Equations	86
4.2.1	Mass Zoning	86
4.2.2	Equations of Motion	87
4.2.3	Conservation of Mass	88
4.2.4	Calculation of Incremental Strain	89
4.2.5	Calculation of Stresses	90
4.2.6	Von Mises Yield Condition	92
4.2.7	Equivalent Plastic Strain, ε^P	92
4.2.8	Artificial Viscosity for Calculating Shocks	93
4.2.9	Navier–Stokes Artificial Viscosity for Stabilizing the Grid	94
4.2.10	Material Internal Energy	96
4.2.11	Calculation of Time Steps, $\Delta t^{n+3/2}$ and Δt^{n+1}	97
4.2.12	Energy Summations (Edit Routine)	97
4.2.13	Principal Stresses (Edit Routine)	98
4.2.14	Calculation of Load, L , on a Given k Line (Edit Routine)	98
4.3	Boundary Conditions	99
4.3.1	Fixed Boundary on the x Axis	99
4.3.2	Fixed Boundary on the y Axis	100
4.3.3	Corner Zone on the x Axis	100
4.3.4	Corner Zone on the y Axis	101
4.3.5	Free Surfaces	102
4.3.6	Discussion	102
4.4	Applications	103

5.	Sliding Interfaces in Two Dimensions	113
5.1	Sliding Interfaces Between Quadrilateral Lagrange Zones	114
5.1.1	Location of Master Points Associated with a Given Slave Point	115
5.1.2	Calculation of the Volume of Sliding Zones Associated with the Slave Grid	115
5.1.3	Advancing a Slave Point f in Time	116
5.1.4	Location of Slave Points Associated with a Given Master Point	120
5.1.5	Advancement in Time of Point j, k on the Master Grid	121
5.1.6	Testing for Penetration of Grids	123
5.1.7	Adjusting the Velocities of All Void Closed Points Where $d < 0$ and Where in the Previous Cycle the Point Was Void Open	124
5.1.8	Relocating Slave Points onto the Master Surface when $d < 0$	126
5.2	Intersecting Slide Lines	126
5.2.1	Acceleration of Points on the Intersection of Two Slide Lines	126
5.2.2	Adjustment for Grid Penetration	127
5.2.3	Relocation of Points when a Void Has Opened	127
6.	Elastic–Plastic Flow in Three Space Dimensions	129
6.1	Fundamental Equations	129
6.1.1	Equations of Motion	129
6.1.2	Conservation of Mass	129
6.1.3	First Law of Thermodynamics	129
6.1.4	Velocity Strains	130
6.1.5	Stress Deviator Tensor	130
6.1.6	Pressure Equation of State	130
6.1.7	Total Stresses	131
6.1.8	Artificial Viscosity for Calculating Shocks	131
6.1.9	Von Mises Yield Condition	131
6.2	Finite Difference Equations for HEMP 3D	131
6.2.1	Mass Zoning	131
6.2.2	Equations of Motion	133
6.2.3	Conservation of Mass	136
6.2.4	Calculation of Incremental Strains	136
6.2.5	Calculation of Stresses	139
6.2.6	Von Mises Yield Condition	140
6.2.7	Plastic Strain	140
6.2.8	Artificial Viscosity for Calculating Shocks	141
6.2.9	Tensor Artificial Viscosity for Stabilizing the Grid	142
6.2.10	Material Internal Energy	145

6.2.11	Time Step Calculations	146
6.3	Boundary Conditions	146
6.4	Check Problems	146
6.4.1	Simple Harmonic Motion	146
6.4.2	Plasticity	149
7.	Sliding Surfaces in Three Dimensions	151
7.1	Calculational Steps to Advance in Time Grid Points on a Sliding Surface	153
7.2	Applications of Sliding Surface Routine	163
7.3	Zone Dimension Change and Subcycling	163
7.3.1	Zone Dimension Change at an Interface in Two Dimensions	163
7.3.2	Zone Dimension Change of an Interface in Three Dimensions	167
7.3.3	Subcycling with Zone Dimension Change in Two Dimensions	169
7.3.4	Example for a Zone Size Change of Two to One	169
8.	Magnetohydrodynamics of HEMP	171
8.1	Finite Difference Scheme for Double Operators	172
8.2	Fundamental Equations of Magnetohydrodynamics	174
8.2.1	Equation of Motion	174
8.2.2	Electromagnetic Field Equations	174
8.2.3	Energy Equation	175
8.2.4	Continuity Equation	176
8.2.5	Constitutive Relations	176
8.3	Difference Equations for Magnetohydrodynamics	176
8.3.1	Equations of Motion	176
8.3.2	Magnetic Diffusion	177
8.3.3	Energy Equations	179
8.3.4	Continuity Equation	182
8.3.5	Time-Step Control	182
8.3.6	Boundary Conditions	183
8.3.7	Sliding Interfaces	183
8.3.8	Check Problems	185
	Appendices	189
A.	Effect of a Second Shock on the Principal Hugoniot	189
B.	Finite Difference Program for One Space Dimension and Time	191
B.1	Fundamental Equations	191
B.2	Finite Difference Equations	192
B.3	Boundary Conditions	194
B.4	Opening and Closing Voids	195

XII Table of Contents

C. A Method for Determining the Plastic Work Hardening Function	197
C.1 Application to 6061-T6 Aluminum.....	199
D. Detonation of a High Explosive for a γ -Law Equation of State	202
E. Magnetic Flux Calculation.....	211
F. Thermal Diffusion Calculation	224
G. Backward Substitution Method for Solving a System of Linear Equations of the Form $A_i H_{i+1} + B_i H_i + C_i H_{i-1} = D_i$	238
References	241
Subject Index	245