

Table of Contents

Preface	VII
Motivation	VII
Aims, Readership and Book Structure	XII
Final Word and Acknowledgments	XIII
Description of Contents by Chapter	XVI
Abbreviations and Notation	XXV

Part I. MODELS: THEORY AND IMPLEMENTATION

1. Definitions and Notation	1
1.1 The Bank Account and the Short Rate	1
1.2 Zero-Coupon Bonds and Spot Interest Rates	3
1.3 Fundamental Interest-Rate Curves	8
1.4 Forward Rates	10
1.5 Interest-Rate Swaps and Forward Swap Rates	13
1.6 Interest-Rate Caps/Floors and Swaptions	15
2. No-Arbitrage Pricing and Numeraire Change	23
2.1 No-Arbitrage in Continuous Time	24
2.2 The Change-of-Numeraire Technique	26
2.3 A Change-of-Numeraire Toolkit	28
2.4 The Choice of a Convenient Numeraire	32
2.5 The Forward Measure	33
2.6 The Fundamental Pricing Formulas	35
2.6.1 The Pricing of Caps and Floors	36
2.7 Pricing Claims with Deferred Payoffs	37
2.8 Pricing Claims with Multiple Payoffs	38
2.9 Foreign Markets and Numeraire Change	40
3. One-factor short-rate models	43
3.1 Introduction and Guided Tour	43
3.2 Classical Time-Homogeneous Short-Rate Models	48
3.2.1 The Vasicek Model	50

3.2.2	The Dothan Model	54
3.2.3	The Cox, Ingersoll and Ross (CIR) Model	56
3.2.4	Affine Term-Structure Models	60
3.2.5	The Exponential-Vasicek (EV) Model	61
3.3	The Hull-White Extended Vasicek Model	63
3.3.1	The Short-Rate Dynamics	64
3.3.2	Bond and Option Pricing	66
3.3.3	The Construction of a Trinomial Tree	69
3.4	Possible Extensions of the CIR Model	72
3.5	The Black-Karasinski Model	73
3.5.1	The Short-Rate Dynamics	74
3.5.2	The Construction of a Trinomial Tree	76
3.6	Volatility Structures in One-Factor Short-Rate Models	77
3.7	Humped-Volatility Short-Rate Models	83
3.8	A General Deterministic-Shift Extension	86
3.8.1	The Basic Assumptions	87
3.8.2	Fitting the Initial Term Structure of Interest Rates	88
3.8.3	Explicit Formulas for European Options	90
3.8.4	The Vasicek Case	91
3.9	The CIR++ Model	93
3.9.1	The Construction of a Trinomial Tree	96
3.9.2	The Positivity of Rates and Fitting Quality	97
3.10	Deterministic-Shift Extension of Lognormal Models	100
3.11	Some Further Remarks on Derivatives Pricing	102
3.11.1	Pricing European Options on a Coupon-Bearing Bond	102
3.11.2	The Monte Carlo Simulation	103
3.11.3	Pricing Early-Exercise Derivatives with a Tree	106
3.11.4	A Fundamental Case of Early Exercise: Bermudan-Style Swaptions	111
3.12	Implied Cap Volatility Curves	114
3.12.1	The Black and Karasinski Model	115
3.12.2	The CIR++ Model	116
3.12.3	The Extended Exponential-Vasicek Model	117
3.13	Implied Swaption Volatility Surfaces	119
3.13.1	The Black and Karasinski Model	120
3.13.2	The Extended Exponential-Vasicek Model	120
3.14	An Example of Calibration to Real-Market Data	121
4.	Two-Factor Short-Rate Models	127
4.1	Introduction and Motivation	127
4.2	The Two-Additive-Factor Gaussian Model G2++	132
4.2.1	The Short-Rate Dynamics	133
4.2.2	The Pricing of a Zero-Coupon Bond	134
4.2.3	Volatility and Correlation Structures in Two-Factor Models	137

4.2.4	The Pricing of a European Option on a Zero-Coupon Bond	143
4.2.5	The Analogy with the Hull-White Two-Factor Model ..	149
4.2.6	The Construction of an Approximating Binomial Tree ..	152
4.2.7	Examples of Calibration to Real-Market Data	156
4.3	The Two-Additive-Factor Extended CIR/LS Model CIR2++ ..	165
4.3.1	The Basic Two-Factor CIR2 Model	166
4.3.2	Relationship with the Longstaff and Schwartz Model (LS)	167
4.3.3	Forward-Measure Dynamics and Option Pricing for CIR2	168
4.3.4	The CIR2++ Model and Option Pricing	168
5.	The Heath-Jarrow-Morton (HJM) Framework	173
5.1	The HJM Forward-Rate Dynamics	175
5.2	Markovianity of the Short-Rate Process	176
5.3	The Ritchken and Sankarasubramanian Framework	177
5.4	The Mercurio and Moraleda Model	181
6.	The LIBOR and Swap Market Models (LFM and LSM) ..	183
6.1	Introduction	183
6.2	Market Models: a Guided Tour	184
6.3	The Lognormal Forward-LIBOR Model (LFM)	192
6.3.1	Some Specifications of the Instantaneous Volatility of Forward Rates	195
6.3.2	Forward-Rate Dynamics under Different Numeraires ..	198
6.4	Calibration of the LFM to Caps and Floors Prices	203
6.4.1	Piecewise-Constant Instantaneous-Volatility Structures ..	206
6.4.2	Parametric Volatility Structures	207
6.4.3	Cap Quotes in the Market	208
6.5	The Term Structure of Volatility	210
6.5.1	Piecewise-Constant Instantaneous Volatility Structures ..	210
6.5.2	Parametric Volatility Structures	215
6.6	Instantaneous Correlation and Terminal Correlation	217
6.7	Swaptions and the Lognormal Forward-Swap Model (LSM) ..	220
6.7.1	Swaptions Hedging	224
6.7.2	Cash-Settled Swaptions	226
6.8	Incompatibility between the LFM and the LSM	227
6.9	The Structure of Instantaneous Correlations	230
6.10	Monte Carlo Pricing of Swaptions with the LFM	233
6.11	Rank-One Analytical Swaption Prices	236
6.12	Rank- r Analytical Swaption Prices	242
6.13	A Simpler LFM Formula for Swaptions Volatilities	246
6.14	A Formula for Terminal Correlations of Forward Rates	249
6.15	Calibration to Swaptions Prices	252

6.16	Connecting Caplet and $S \times 1$ -Swaption Volatilities	254
6.17	Forward and Spot Rates over Non-Standard Periods	261
6.17.1	Drift Interpolation	262
6.17.2	The Bridging Technique	264
6.18	Including the Caplet Smile in the LFM	266
6.18.1	A Mini-tour on the Smile Problem	266
6.18.2	Modeling the Smile	270
6.18.3	The Shifted-Lognormal Case	271
6.18.4	The Constant Elasticity of Variance (CEV) Model	273
6.18.5	A Mixture-of-Lognormals Model	276
6.18.6	Shifting the Lognormal-Mixture Dynamics	280
7.	Cases of Calibration of the LIBOR Market Model	283
7.1	The Inputs	284
7.2	Joint Calibration with Piecewise-Constant Volatilities as in TABLE 5	284
7.2.1	Instantaneous Correlations: Narrowing the Angles	288
7.2.2	Instantaneous Correlations: Fixing the Angles to Typ- ical Values	290
7.2.3	Instantaneous Correlations: Fixing the Angles to Atyp- ical Values	292
7.2.4	Instantaneous Correlations: Collapsing to One Factor	293
7.3	Joint Calibration with Parameterized Volatilities as in For- mulation 7	295
7.3.1	Formulation 7: Narrowing the Angles	297
7.3.2	Formulation 7: Calibrating only to Swaptions	300
7.4	Exact Swaptions Calibration with Volatilities as TABLE 1	303
7.4.1	Some Numerical Results	309
7.5	Conclusions: Where Now?	314
8.	Monte Carlo Tests for LFM Analytical Approximations	317
8.1	The Specification of Rates	317
8.2	The “Testing Plan” for Volatilities	318
8.3	Test Results for Volatilities	321
8.3.1	Case (1): Constant Instantaneous Volatilities	322
8.3.2	Case (2): Volatilities as Functions of Time to Maturity	329
8.3.3	Case (3): Humped and Maturity-Adjusted Instanta- neous Volatilities Depending only on Time to Matu- rity, Typical Rank-Two Correlations	334
8.4	The “Testing Plan” for Terminal Correlations	345
8.5	Test Results for Terminal Correlations	353
8.5.1	Case (i): Humped and Maturity-Adjusted Instanta- neous Volatilities Depending only on Time to Matu- rity, Typical Rank-Two Correlations	353

8.5.2	Case (ii): Constant Instantaneous Volatilities, Typical Rank-Two Correlations.	355
8.5.3	Case (iii): Humped and Maturity-Adjusted Instantaneous Volatilities Depending only on Time to Maturity, Some Negative Rank-Two Correlations.	359
8.5.4	Case (iv): Constant Instantaneous Volatilities, Some Negative Rank-Two Correlations.	363
8.5.5	Case (v): Constant Instantaneous Volatilities, Perfect Correlations, Upwardly Shifted Φ 's	365
8.6	Test Results: Stylized Conclusions	367
9.	Other Interest-Rate Models	369
9.1	Brennan and Schwartz's Model	369
9.2	Balduzzi, Das, Foresi and Sundaram's Model	370
9.3	Flesaker and Hughston's Model	371
9.4	Rogers's Potential Approach	373
9.5	Markov Functional Models.	373

Part II. PRICING DERIVATIVES IN PRACTICE

10.	Pricing Derivatives on a Single Interest-Rate Curve	377
10.1	In-Advance Swaps	378
10.2	In-Advance Caps	379
10.2.1	A First Analytical Formula (LFM)	380
10.2.2	A Second Analytical Formula (G2++)	380
10.3	Autocaps	381
10.4	Caps with Deferred Caplets	382
10.4.1	A First Analytical Formula (LFM)	382
10.4.2	A Second Analytical Formula (G2++)	383
10.5	Ratchets (One-Way Floaters)	384
10.6	Constant-Maturity Swaps (CMS)	385
10.6.1	CMS with the LFM	385
10.6.2	CMS with the G2++ Model	386
10.7	The Convexity Adjustment and Applications to CMS	386
10.7.1	Natural and Unnatural Time Lags	386
10.7.2	The Convexity-Adjustment Technique.	387
10.7.3	Deducing a Simple Lognormal Dynamics from the Adjustment	391
10.7.4	Application to CMS	392
10.7.5	Forward Rate Resetting Unnaturally and Average-Rate Swaps	393
10.8	Captions and Floortions	395
10.9	Zero-Coupon Swaptions	395
10.10	Eurodollar Futures	399

10.10.1	The Shifted Two-Factor Vasicek G2++ Model.....	400
10.10.2	Eurodollar Futures with the LFM.....	402
10.11	LFM Pricing with “In-Between” Spot Rates.....	402
10.11.1	Accrual Swaps	403
10.11.2	Trigger Swaps	406
10.12	LFM Pricing with Early Exercise and Possible Path Dependence	408
10.13	LFM: Pricing Bermudan Swaptions	412
10.13.1	Longstaff and Schwartz’s Approach	413
10.13.2	Carr and Yang’s Approach.....	415
10.13.3	Andersen’s Approach.....	416
11.	Pricing Derivatives on Two Interest-Rate Curves	421
11.1	The Attractive Features of G2++ for Multi-Curve Payoffs ...	421
11.1.1	The Model	421
11.1.2	Interaction Between Models of the Two Curves “1” and “2”	424
11.1.3	The Two-Models Dynamics under a Unique Convenient Forward Measure.....	425
11.2	Quanto Constant-Maturity Swaps	427
11.2.1	Quanto CMS: The Contract	427
11.2.2	Quanto CMS: The G2++ Model	429
11.2.3	Quanto CMS: Quanto Adjustment.....	435
11.3	Differential Swaps	437
11.3.1	The Contract	437
11.3.2	Differential Swaps with the G2++ Model.....	438
11.3.3	A Market-Like Formula	440
11.4	Market Formulas for Basic Quanto Derivatives	440
11.4.1	The Pricing of Quanto Caplets/Floorlets	440
11.4.2	The Pricing of Quanto Caps/Floors.....	443
11.4.3	The Pricing of Differential Swaps	444
11.4.4	The Pricing of Quanto Swaptions.....	444
12.	Pricing Equity Derivatives under Stochastic Rates	453
12.1	The Short Rate and Asset-Price Dynamics.....	453
12.1.1	The Dynamics under the Forward Measure	456
12.2	The Pricing of a European Option on the Given Asset	458
12.3	A More General Model	459
12.3.1	The Construction of an Approximating Tree for r ...	460
12.3.2	The Approximating Tree for S	462
12.3.3	The Two-Dimensional Tree	463

Part III. APPENDICES

A. A Crash Introduction to Stochastic Differential Equations	469
A.1 From Deterministic to Stochastic Differential Equations	469
A.2 Ito's Formula	476
A.3 Discretizing SDEs for Monte Carlo: Euler and Milstein Schemes	478
A.4 Examples	480
A.5 Two Important Theorems	482
B. A Useful Calculation	485
C. Approximating Diffusions with Trees	487
D. Talking to the Traders	493
References	501
Index	509