## TABLE OF CONTENTS

| I. General facts about the method, purpose of the paper                              | 1  |
|--|----|
| 1. Limit theorems for Markov chains  | 1  |
| 2. Stochastic properties of dynamical systems  | 2  |
| 3. Historical background to the method   | 3  |
| 4. Purpose of the paper  | 4  |
| II. The central limit theorems for Markov chains                                     | 6  |
| 1. The concept of quasi-compact operator   | 6  |
| 2. Conditions $\mathcal{H}[m]$ and $\widehat{\mathcal{D}}$ , notations $\mathcal{N}$ | 8  |
| 3. Statements of the central limit theorems  | 11 |
| III. Quasi-compact operators of diagonal type and perturbations                      | 14 |
| 1. Definition, properties  | 14 |
| 2. A perturbation theorem  | 18 |
| IV. First properties of Fourier kernels, application                                 | 23 |
| 1. Properties of the Fourier kernels   | 23 |
| 2. Central limit theorem: intermediate result  | 27 |
| V. Peripheral eigenvalues of Fourier kernels   | 31 |
| 1. Eigenvalues of $Q(t)$ of modulus 1  | 31 |
| 2. Peripheral eigenvalues of $Q(t)$ for small $ t $                                  | 34 |
| VI. Proofs of Theorems A, B, C   | 38 |
| 1. Conditions $\mathcal{H}''[m]$ . Central limit theorem (Theorem A)                 | 38 |
| 2. Development of the characteristic function  | 38 |
| 3. Central limit theorem with a rate of convergence (Theorem B)                      | 39 |
| 4. Local central limit theorem (Theorem C)   | 41 |
| VII. Renewal theorem for Markov chains (Theorem D)                                   | 43 |
| 1. Statements  | 43 |
| 2. Proof of Theorem VII.2  | 44 |
| VIII. Large deviations for Markov chains (Theorem E)                                 | 49 |
| 1. Statement of the main result  | 49 |
| 2. Properties of the Laplace kernels, function c                                     | 50 |
| 3. Logarithmic estimate: Theorem E-(i)-(ii)  | 52 |
| 4. Probability of a large deviation: Theorem E-(iii)                                 | 54 |
| 5. Additional statements   | 58 |

| IX. Ergodic properties for Markov chains                    | 60  |
|---|-----|
| X. Markov chains associated with Lipschitz kernels          | 63  |
| 1. General facts, contraction properties                    | 63  |
| 2. Invariant distributions and quasi-compactness            | 64  |
| 3. Laplace kernels  | 70  |
| 4. Products of invertible random matrices                   | 75  |
| 5. Products of positive random matrices                     | 78  |
| 6. Autoregresive processes                                  | 79  |
| XI. Stochastic properties of dynamical systems              | 81  |
| 1. Statements   | 81  |
| 2. $	au$ -invariant distribution, relativized Markov kernel | 84  |
| 3. Proofs of the limit theorems                             | 86  |
| XII. Expanding maps   | 89  |
| 1. Piecewise expanding maps of the interval                 | 89  |
| 2. Subshifts and transfer operators                         | 93  |
| XIII. Proofs of some statements in Probability Theory       | 99  |
| 1. Example of a two state Markov chain                      | 99  |
| 2. Proof of Lemma IV-5                                      | 101 |
| 3. Large deviations lemma                                   | 102 |
| XIV. Functional analysis results on quasi-compactness       | 104 |
| 1. A sufficient condition for quasi-compactness             | 104 |
| 2. Proof of the perturbation theorem (Theorem III.8)        | 111 |
| Generalization to the non-ergodic case, by L. Hervé         | 115 |
| References  | 141 |

145

Indexes