

Table of Contents

0	Paradoxical decompositions	1
0.1	The Banach–Tarski paradox	1
0.2	Tarski’s theorem	7
0.3	Notes and comments	14
1	Amenable, locally compact groups	17
1.1	Invariant means on locally compact groups	17
1.2	Hereditary properties	23
1.3	Day’s fixed point theorem	29
1.4	Representations on Hilbert space	30
1.5	Notes and comments	34
2	Amenable Banach algebras	37
2.1	Johnson’s theorem	37
2.2	Virtual and approximate diagonals	43
2.3	Hereditary properties	46
2.4	Hochschild cohomology	55
2.5	Notes and comments	60
3	Examples of amenable Banach algebras	63
3.1	Banach algebras of compact operators	63
3.2	A commutative, radical, amenable Banach algebra	71
3.3	Notes and comments	80
4	Amenability-like properties	83
4.1	Super-amenability	83
4.2	Weak amenability	87
4.3	Biprojectivity and biflatness	95
4.4	Connes-amenability	108
4.5	Notes and comments	115
5	Banach homology	119
5.1	Projectivity	119
5.2	Resolutions and Ext-groups	123
5.3	Flatness and injectivity	133
5.4	Notes and comments	138

6	C^*- and W^*-algebras	141
6.1	Amenable W^* -algebras	142
6.2	Injective W^* -algebras	148
6.3	Tensor products of C^* - and W^* -algebras	154
6.4	Semidiscrete W^* -algebras	162
6.5	Normal, virtual diagonals	177
6.6	Notes and comments	187
7	Operator amenability	191
7.1	Bounded approximate identities for Fourier algebras	191
7.2	(Non-)amenability of Fourier algebras	195
7.3	Operator amenable operator Banach algebras	199
7.4	Operator amenability of Fourier algebras	200
7.5	Operator amenability of C^* -algebras	205
7.6	Notes and comments	205
8	Geometry of spaces of homomorphisms	209
8.1	Infinite-dimensional differential geometry	209
8.2	Spaces of homomorphisms	217
8.3	Notes and comments	219
	Open problems	221
A	Abstract harmonic analysis	231
A.1	Convolution of measures and functions	231
A.2	Invariant subspaces of $L^\infty(G)$	235
A.3	Regular representations on $L^p(G)$ and associated algebras	237
A.4	Notes and comments	241
B	Tensor products	243
B.1	The algebraic tensor product	243
B.2	Banach space tensor products	246
B.2.1	The injective tensor product	247
B.2.2	The projective tensor product	250
B.3	The Hilbert space tensor product	253
B.4	Notes and comments	254
C	Banach space properties	255
C.1	Approximation properties	255
C.2	The Radon–Nikodým property	258
C.3	Local theory of Banach spaces	259
C.4	Notes and comments	263

D Operator spaces	265
D.1 Abstract and concrete operator spaces	265
D.2 Completely bounded maps	267
D.3 Tensor products of operator spaces	269
D.4 Operator Banach algebras	271
D.5 Notes and comments	273
List of symbols	275
References	281
Index	289