

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1	The Calculus of Variations . . . . .	1
2	Optimal Control . . . . .	5
3	Numerical Methods for Optimal Control Problems . . . . .	7
<b>2</b>	<b>Estimates on Solutions to Differential Equations and Their Approximations</b>	<b>13</b>
1	Linear Approximations . . . . .	13
2	Lagrangian, Hamiltonian and Reduced Gradients . . . . .	19
<b>3</b>	<b>First Order Method</b>	<b>27</b>
1	Introduction . . . . .	27
2	Representation of Functional Directional Derivatives . . . . .	31
3	Relaxed Controls . . . . .	32
4	The Algorithm . . . . .	34
5	Convergence Properties of the Algorithm . . . . .	38
6	Proof of the Convergence Theorem, etc. . . . .	41
7	Concluding Remarks . . . . .	52
<b>4</b>	<b>Implementation</b>	<b>55</b>
1	Implementable Algorithm . . . . .	55
1.1	Second Order Correction To the Line Search . . . . .	65
1.2	Resetting the Penalty Parameter . . . . .	66
2	Semi-Infinite Programming Problem . . . . .	66
3	Numerical Examples . . . . .	68
<b>5</b>	<b>Second Order Method</b>	<b>81</b>
1	Introduction . . . . .	81
2	Function Space Algorithm . . . . .	84
3	Semi-Infinite Programming Method . . . . .	86
4	Bounding the Number of Constraints . . . . .	92
4.1	Some Remarks on Direction Finding Subproblems . . . . .	94
4.2	The Nonlinear Programming Problem . . . . .	98

4.3	The Watchdog Technique for Redundant Constraints	107
4.4	Two-Step Superlinear Convergence . . . . .	121
4.5	Numerical Experiments . . . . .	125
5	Concluding Remarks . . . . .	127
<b>6</b>	<b>Runge-Kutta Based Procedure for Optimal Control of Differential — Algebraic Equations</b>	<b>129</b>
1	Introduction . . . . .	129
2	The Method . . . . .	133
2.1	Implicit Runge-Kutta Methods . . . . .	134
2.2	Calculation of the Reduced Gradients . . . . .	137
3	Implementation of the Implicit Runge-Kutta Method . . . . .	144
3.1	Simplified Newton Iterations . . . . .	144
3.2	Stopping Criterion for the Newton Method . . . . .	145
3.3	Stepsize Selection . . . . .	146
4	Numerical Experiments . . . . .	151
5	Some Remarks on Integration and Optimization Accuracies	164
6	Concluding Remarks . . . . .	166
<b>A</b>	<b>A Primal Range-Space Method for Piecewise-Linear Quadratic Programming</b>	<b>169</b>
A.1	Software Implementation . . . . .	169
A.2	A Range-Space Method—Introduction . . . . .	170
A.3	The Basic Method . . . . .	171
A.4	Efficient Implementation . . . . .	175
A.4.1	Adding a Bound to the Working Set . . . . .	178
A.4.2	Deleting a Bound from the Working Set . . . . .	182
A.4.3	Adding a Vector $a$ to the Working Set . . . . .	184
A.4.4	Deleting a Vector $a$ from the Working Set . . . . .	186
A.5	Computation of the Lagrange Multipliers... . . . . .	187
A.6	Modifications and Extensions . . . . .	188
A.7	Numerical Experiments . . . . .	191
<b>References</b>		<b>197</b>
<b>List of Symbols</b>		<b>209</b>
<b>Subject Index</b>		<b>213</b>