Contents

Introduction

Introduction	1
CHAPTER I	-
Basic Structural Tricks and Examples	5
1. Algebras and their Defining Relations	5
2. Skew Polynomial Rings	9
3. Z-filtrations and their Associated Graded Structures	13
4. Homogenization and Dehomogenization of Z-graded Rings	25
5. Some Algebras: Classical and Modern	28
CHAPTER II	
Gröbner Bases in Associative Algebras	33
1. (Left) Monomial Orderings and (Left) Admissible Systems	34
2. (Left) Quasi-zero Elements and a (Left) Division Algorithm	38
3. (Left) Gröbner Bases in a (Left) Admissible System	42
4. (Left) Gröbner Bases in Various Contexts	45
5. (Left) S-elements and Buchberger Theorem	48
6. (Left) Dickson Systems and (Left) G-Noetherian Algebras	54
7. Solvable Polynomial Algebras	58
8. No (Left) Monomial Ordering Existing on $\Delta(k[x_1,,x_n])$	
with $chark > 0$	61
CHAPTER III	
Gröbner Bases and Basic Algebraic-Algorithmic	
Structures	67
1. PBW Bases of Finitely Generated Algebras	68
2. Quadric Solvable Polynomial Algebras	73
3. Associated Homogeneous Defining Relations of Algebras	81
4. A Remark on Recognizable Properties of Algebras	
via Gröbner Bases	89

viii Contents

OHADTED IV	
CHAPTER IV Filtered-Graded Transfer of Gröbner Bases	91
1. Filtered-Graded Transfer of (Left) Admissible Systems	92
2. Filtered-Graded Transfer of (Left) Gröbner Bases	97
3. Filtered-Graded Transfer of (Left) Dickson Systems	100
4. Filtered-Graded Transfer Applied to Quadric Solvable	
Polynomial Algebras	103
CHAPTED V	
CHAPTER V GK-dimension of Modules over Quadric Solvable	
Polynomial Algebras and Elimination of Variables	107
1. Gröbner Bases in Homogeneous Solvable Polynomial Algebras	108
2. The Hilbert Function of A/L	110
·	112
3. The Hilbert Polynomial of A/L4. GK-dimension Computation and Elimination of Variables	112
(Homogeneous Case)	115
5. GK-dimension Computation and Elimination of Variables	110
(Linear Case)	119
6. The \succeq_{gr} -filtration on a Quadric Solvable Polynomial Algebra	124
7. GK-dimension Computation and Elimination of Variables	121
(General Quadric Case)	126
8. Finite Dimensional Cyclic Modules	130
6. Finite Diffensional Cyclic Modules	100
CHAPTER VI	
Multiplicity Computation of Modules over Quadric	
Solvable Polynomial Algebras	133
1. The Multiplicity $e(M)$ of a Module M	134
2. Computation of $e(M)$	135
3. Computation of $GK.dim(M \otimes_k N)$ and $e(M \otimes_k N)$	144
4. An Application to $A_n(q_1,,q_n)$	148
CHAPTER VII	
(∂-)Holonomic Modules and Functions over Quadric	
Solvable Polynomial Algebras	153
1. Some Operator Algebras	154
2. Holonomic Functions	156
3. Automatic Proving of Holonomic Function Identities	162
4. Extension/Contraction of the ∂ -finiteness	164
5. The 2-holonomicity	168

Contents ix

CHAPTER VIII	
Regularity and K_0 -group of Quadric Solvable	
Polynomial Algebras	175
1. Tame Case: A is Auslander Regular with $K_0(A) \cong \mathbb{Z}$	176
2. The \succeq_{gr} -filtration on Modules	177
3. General Case: $\operatorname{gl.dim} A \leq n$	181
4. General Case: $K_0(A) \cong \mathbb{Z}$	186
References	187
Index	195