

Contents

Preface	v
1 Let's Count!	1
1.1 A Party	1
1.2 Sets and the Like	4
1.3 The Number of Subsets	9
1.4 The Approximate Number of Subsets	14
1.5 Sequences	15
1.6 Permutations	17
1.7 The Number of Ordered Subsets	19
1.8 The Number of Subsets of a Given Size	20
2 Combinatorial Tools	25
2.1 Induction	25
2.2 Comparing and Estimating Numbers	30
2.3 Inclusion-Exclusion	32
2.4 Pigeonholes	34
2.5 The Twin Paradox and the Good Old Logarithm	37
3 Binomial Coefficients and Pascal's Triangle	43
3.1 The Binomial Theorem	43
3.2 Distributing Presents	45
3.3 Anagrams	46
3.4 Distributing Money	48

3.5	Pascal's Triangle	49
3.6	Identities in Pascal's Triangle	50
3.7	A Bird's-Eye View of Pascal's Triangle	54
3.8	An Eagle's-Eye View: Fine Details	57
4	Fibonacci Numbers	65
4.1	Fibonacci's Exercise	65
4.2	Lots of Identities	68
4.3	A Formula for the Fibonacci Numbers	71
5	Combinatorial Probability	77
5.1	Events and Probabilities	77
5.2	Independent Repetition of an Experiment	79
5.3	The Law of Large Numbers	80
5.4	The Law of Small Numbers and the Law of Very Large Numbers	83
6	Integers, Divisors, and Primes	87
6.1	Divisibility of Integers	87
6.2	Primes and Their History	88
6.3	Factorization into Primes	90
6.4	On the Set of Primes	93
6.5	Fermat's "Little" Theorem	97
6.6	The Euclidean Algorithm	99
6.7	Congruences	105
6.8	Strange Numbers	107
6.9	Number Theory and Combinatorics	114
6.10	How to Test Whether a Number is a Prime?	117
7	Graphs	125
7.1	Even and Odd Degrees	125
7.2	Paths, Cycles, and Connectivity	130
7.3	Eulerian Walks and Hamiltonian Cycles	135
8	Trees	141
8.1	How to Define Trees	141
8.2	How to Grow Trees	143
8.3	How to Count Trees?	146
8.4	How to Store Trees	148
8.5	The Number of Unlabeled Trees	153
9	Finding the Optimum	157
9.1	Finding the Best Tree	157
9.2	The Traveling Salesman Problem	161
10	Matchings in Graphs	165

10.1 A Dancing Problem	165
10.2 Another matching problem	167
10.3 The Main Theorem	169
10.4 How to Find a Perfect Matching	171
11 Combinatorics in Geometry	179
11.1 Intersections of Diagonals	179
11.2 Counting regions	181
11.3 Convex Polygons	184
12 Euler's Formula	189
12.1 A Planet Under Attack	189
12.2 Planar Graphs	192
12.3 Euler's Formula for Polyhedra	194
13 Coloring Maps and Graphs	197
13.1 Coloring Regions with Two Colors	197
13.2 Coloring Graphs with Two Colors	199
13.3 Coloring graphs with many colors	202
13.4 Map Coloring and the Four Color Theorem	204
14 Finite Geometries, Codes, Latin Squares, and Other Pretty Creatures	211
14.1 Small Exotic Worlds	211
14.2 Finite Affine and Projective Planes	217
14.3 Block Designs	220
14.4 Steiner Systems	224
14.5 Latin Squares	229
14.6 Codes	232
15 A Glimpse of Complexity and Cryptography	239
15.1 A Connecticut Class in King Arthur's Court	239
15.2 Classical Cryptography	242
15.3 How to Save the Last Move in Chess	244
15.4 How to Verify a Password—Without Learning it	246
15.5 How to Find These Primes	246
15.6 Public Key Cryptography	247
16 Answers to Exercises	251
Index	287