

Contents

1. Mathematical Preliminaries	1
1.1 Introduction	1
1.2 Characteristics and Classification	8
1.3 Orthogonal Functions	13
1.4 Sturm–Liouville Boundary Value Problems	16
1.5 Legendre Polynomials	18
1.6 Bessel Functions	24
1.7 Results from Complex Analysis	29
1.8 Generalised Functions and the Delta Function	34
1.8.1 Definition and Properties of a Generalised Function	35
1.8.2 Differentiation Across Discontinuities	41
1.8.3 The Fourier Transform of Generalised Functions	42
1.8.4 Convolution of Generalised Functions	44
1.8.5 The Discrete Representation of the Delta Function	46
2. Separation of the Variables	49
2.1 Introduction	49
2.2 The Wave Equation	50
2.3 The Heat Equation	56
2.4 Laplace's Equation	61
2.5 Homogeneous and Non-homogeneous Boundary Conditions	66
2.6 Separation of variables in other coordinate systems	77
3. First-order Equations and Hyperbolic Second-order Equations	95
3.1 Introduction	95
3.2 First-order equations	95
3.3 Introduction to d'Alembert's Method	101

3.4 d'Alembert's General Solution	105
3.5 Characteristics	108
3.6 Semi-infinite Strings	117
4. Integral Transforms.....	123
4.1 Introduction	123
4.2 Fourier Integrals	124
4.3 Application to the Heat Equation	130
4.4 Fourier Sine and Cosine Transforms.....	133
4.5 General Fourier Transforms	136
4.6 Laplace transform	142
4.7 Inverting Laplace Transforms	144
4.8 Standard Transforms.....	151
4.9 Use of Laplace Transforms to Solve Partial Differential Equations	157
5. Green's Functions	163
5.1 Introduction	163
5.2 Green's Functions for the Time-independent Wave Equation ..	168
5.3 Green's Function Solution to the Three-dimensional Inhomogeneous Wave Equation.....	176
5.4 Green's Function Solutions to the Inhomogeneous Helmholtz and Schrödinger Equations: An Introduction to Scattering Theory	180
5.5 Green's Function Solution to Maxwell's Equations and Time-dependent Problems	194
5.6 Green's Functions and Optics: Kirchhoff Diffraction Theory ..	202
5.7 Approximation Methods and the Born Series	208
5.8 Green's Function Solution to the Diffusion Equation	217
5.9 Green's Function Solution to the Laplace and Poisson Equations	221
5.10 Discussion	223
A. Solutions of Exercises	225
Bibliography	293
Index	297