

# Contents

<b>Preface</b>	<b>v</b>
<b>About the Authors</b>	<b>xi</b>
<b>A Diagram for the Book</b>	<b>xvii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Generalized Coordinates and Newton–Euler Balance . . . . .	2
1.2 Hamilton’s Principle . . . . .	6
1.3 The Lagrange–d’Alembert Principle . . . . .	11
1.4 The Vertical Rolling Disk . . . . .	15
1.5 The Falling Rolling Disk . . . . .	21
1.6 The Knife Edge . . . . .	23
1.7 The Chaplygin Sleigh . . . . .	25
1.8 The Heisenberg System . . . . .	29
1.9 The Rigid Body . . . . .	34
1.10 The Roller Racer . . . . .	42
1.11 The Rattleback . . . . .	43
1.12 The Toda Lattice . . . . .	45
<b>2 Mathematical Preliminaries</b>	<b>49</b>
2.1 Vector Fields, Flows, and Differential Equations . . . . .	49
2.2 Differentiable Manifolds . . . . .	62
2.3 Stability . . . . .	71

2.4	Center Manifold . . . . .	74
2.5	Differential Forms . . . . .	79
2.6	Lie Derivatives . . . . .	86
2.7	Stokes's Theorem, Riemannian Manifolds, Distributions . . . . .	93
2.8	Lie Groups . . . . .	97
2.9	Fiber Bundles and Connections . . . . .	105
<b>3</b>	<b>Basic Concepts in Geometric Mechanics</b>	<b>119</b>
3.1	Symplectic and Poisson Manifolds and Hamiltonian Flows . . . . .	120
3.2	Cotangent Bundles . . . . .	123
3.3	Lagrangian Mechanics and Variational Principles . . . . .	124
3.4	Mechanical Systems with External Forces . . . . .	128
3.5	Lie–Poisson Brackets and the Rigid Body . . . . .	130
3.6	The Euler–Poincaré Equations . . . . .	134
3.7	Momentum Maps . . . . .	136
3.8	Symplectic and Poisson Reduction . . . . .	139
3.9	A Particle in a Magnetic Field . . . . .	142
3.10	The Mechanical Connection . . . . .	144
3.11	The Lagrange–Poincaré Equations . . . . .	146
3.12	The Energy–Momentum Method . . . . .	150
3.13	Coupled Planar Rigid Bodies . . . . .	158
3.14	Phases and Holonomy, the Planar Skater . . . . .	167
<b>4</b>	<b>Introduction to Aspects of Geometric Control Theory</b>	<b>175</b>
4.1	Nonlinear Control Systems . . . . .	175
4.2	Controllability and Accessibility . . . . .	177
4.3	Representation of System Trajectories . . . . .	182
4.4	Averaging and Trajectory Planning . . . . .	190
4.5	Stabilization . . . . .	194
4.6	Hamiltonian and Lagrangian Control Systems . . . . .	198
<b>5</b>	<b>Nonholonomic Mechanics</b>	<b>207</b>
5.1	Equations of Motion . . . . .	211
5.2	The Lagrange–d'Alembert Principle . . . . .	216
5.3	Projected Connections and Newton's Law . . . . .	221
5.4	Systems with Symmetry . . . . .	223
5.5	The Momentum Equation . . . . .	228
5.6	Examples of the Nonholonomic Momentum Map . . . . .	238
5.7	More General Nonholonomic Systems with Symmetries . . . . .	248
5.8	Poisson Geometry of Nonholonomic Systems . . . . .	254
<b>6</b>	<b>Control of Mechanical and Nonholonomic Systems</b>	<b>277</b>
6.1	Background in Kinematic Nonholonomic Control Systems . . . . .	277
6.2	Stabilization of the Heisenberg System . . . . .	285
6.3	Stabilization of a Generalized Heisenberg System . . . . .	291

6.4	Controllability, Accessibility, and Stabilizability . . . . .	301
6.5	Smooth Stabilization to a Manifold . . . . .	304
6.6	Nonsmooth Stabilization . . . . .	308
6.7	Nonholonomic Systems on Riemannian Manifolds . . . . .	319
<b>7</b>	<b>Optimal Control</b>	<b>329</b>
7.1	Variational Nonholonomic Problems . . . . .	329
7.2	Optimal Control and the Maximum Principle . . . . .	336
7.3	Variational Nonholonomic Systems and Optimal Control . . . . .	339
7.4	Kinematic Sub-Riemannian Optimal Control Problems . . . . .	342
7.5	Optimal Control and a Particle in a Magnetic Field . . . . .	353
7.6	Optimal Control of Mechanical Systems . . . . .	358
<b>8</b>	<b>Stability of Nonholonomic Systems</b>	<b>367</b>
8.1	The Nonholonomic Energy-Momentum Method . . . . .	367
8.2	Overview . . . . .	370
8.3	The Pure Transport Case. . . . .	372
8.4	The Nonpure Transport Case . . . . .	377
8.5	General Case—the Lyapunov–Malkin Method . . . . .	387
8.6	Euler–Poincaré–Suslov Equations . . . . .	392
<b>9</b>	<b>Energy-Based Methods for Stabilization</b>	<b>399</b>
9.1	Controlled Lagrangian Methods . . . . .	399
9.2	Feedback Design and Matching . . . . .	402
9.3	Stabilization of a Class of Nonholonomic Systems . . . . .	410
9.4	Averaging for Controlled Lagrangian Systems . . . . .	415
9.5	Dynamic Nonholonomic Averaging . . . . .	435
	<b>References</b>	<b>439</b>
	<b>Index</b>	<b>471</b>