Table of Contents

Chapter I. Balance Laws 1

1.1 Formulation of the Balance Law 2
1.2 Reduction to Field Equations 2
1.3 Change of Coordinates 6
1.4 Systems of Balance Laws 7
1.5 Companion Systems of Balance Laws 8
1.6 Weak and Shock Fronts 10
1.7 Survey of the Theory of BV Functions 11
1.8 BV Solutions of Systems of Balance Laws 15
1.9 Rapid Oscillations and the Stabilizing Effect of Companion Balance Laws 17
1.10 Notes 18

Chapter II. Introduction to Continuum Physics 19

2.1 Bodies and Motions 19
2.2 Balance Laws in Continuum Physics 21
2.3 The Balance Laws of Continuum Thermomechanics 23
2.4 Material Frame Indifference 26
2.5 Thermoelasticity 27
2.6 Thermoviscoelasticity 32
2.7 Notes 34

Chapter III. Hyperbolic Systems of Balance Laws 37

3.1 Hyperbolicity 37
3.2 Entropy-Entropy Flux Pairs 38
3.3 Examples of Hyperbolic Systems of Balance Laws 39
3.4 Notes 45

Chapter IV. The Initial-Value Problem: Admissibility of Solutions 49

4.1 The Initial-Value Problem 49
4.2 The Burgers Equation and Nonuniqueness of Weak Solutions 51
4.3 Entropies and Admissible Solutions 52
4.4 The Vanishing Viscosity Approach 54

4.5 Initial-Boundary-Value Problems 58
4.6 Notes 59

Chapter V. Entropy and the Stability of Classical Solutions 61
5.1 Convex Entropy and the Existence of Classical Solutions 61
5.2 Convex Entropy and the Stability of Classical Solutions 66
5.3 Partially Convex Entropies and Involutions 68
5.4 Notes 82

Chapter VI. The L^1 Theory of the Scalar Conservation Law 83
6.1 The Initial-Value Problem: Perseverance and Demise of Classical Solutions 83
6.2 Admissible Weak Solutions and Their Stability Properties 85
6.3 The Method of Vanishing Viscosity 91
6.4 Solutions as Trajectories of a Contraction Semigroup 95
6.5 The Layering Method 101
6.6 A Kinetic Formulation 105
6.7 Relaxation 110
6.8 The L^1 Theory for Systems of Balance Laws 116
6.9 Notes 117

Chapter VII. Hyperbolic Systems of Balance Laws in One-Space Dimension 121
7.1 Balance Laws in One-Space Dimension 121
7.2 Hyperbolicity and Strict Hyperbolicity 124
7.3 Riemann Invariants 127
7.4 Entropy-Entropy Flux Pairs 131
7.5 Genuine Nonlinearity and Linear Degeneracy 134
7.6 Simple Waves 135
7.7 Breakdown of Classical Solutions 139
7.8 Weak Solutions 143
7.9 Notes 144

Chapter VIII. Admissible Shocks 147
8.1 Strong Shocks, Weak Shocks, and Shocks of Moderate Strength 147
8.2 The Hugoniot Locus 149
8.3 The Lax Shock Admissibility Criterion 154
8.4 The Liu Shock Admissibility Criterion 158
8.5 The Entropy Shock Admissibility Criterion 160
8.6 Viscous Shock Profiles 163
8.7 Notes 172

Chapter IX. Admissible Wave Fans and the Riemann Problem 175
9.1 Self-similar Solutions and the Riemann Problem 175
9.2 Wave Fan Admissibility Criteria 178
9.3 Solution of the Riemann Problem with Admissible Shocks 179
9.4 The Entropy Rate Admissibility Criterion 188
9.5 Viscous Wave Fans 194
9.6 Interaction of Wave Fans 197
9.7 Notes 200

Chapter X. Generalized Characteristics 203
10.1 BV Solutions 203
10.2 Generalized Characteristics 204
10.3 Extremal Backward Characteristics 206
10.4 Notes 208

Chapter XI. Genuinely Nonlinear Scalar Conservation Laws 209
11.1 Admissible BV Solutions and Generalized Characteristics 210
11.2 The Spreading of Rarefaction Waves 213
11.3 Regularity of Solutions 214
11.4 Divides, Invariants and the Lax Formula 218
11.5 Decay of Solutions Induced by Entropy Dissipation 221
11.6 Spreading of Characteristics and Development of N-Waves 224
11.7 Confinement of Characteristics and Formation of Sawtoothed Profiles 225
11.8 Comparison Theorems and L^1 Stability 227
11.9 Notes 235

Chapter XII. Genuinely Nonlinear Systems of Two Conservation Laws ... 239
12.1 Notation and Assumptions 239
12.2 Entropy-Entropy Flux Pairs 241
12.3 Local Structure of Solutions 243
12.4 Propagation of Riemann Invariants Along Extremal Backward Characteristics 246
12.5 Bounds on Solutions 263
12.6 Spreading of Rarefaction Waves 274
12.7 Regularity of Solutions 279
12.8 Initial Data in L^1 281
12.9 Initial Data with Compact Support 285
12.10 Periodic Solutions 291
12.11 Notes 295

Chapter XIII. The Random Choice Method 299
13.1 The Construction Scheme 299

13.2 Compactness and Consistency 302
13.3 Wave Interactions, Approximate Conservation Laws 307
13.4 The Glimm Functional 312
13.5 Bounds on the Total Variation 317
13.6 Bounds on the Supremum 318
13.7 Wave Partitioning 321
13.8 Inhomogeneous Systems of Balance Laws 324
13.9 Breakdown of Weak Solutions 330
13.10 Notes 331

Chapter XIV. The Front Tracking Method and Standard Riemann Semigroups 337
14.1 The Scalar Conservation Law 338
14.2 Front Tracking for Systems of Conservation Laws 340
14.3 The Global Wave Pattern 344
14.4 Approximate Solutions 346
14.5 Bounds on the Total Variation 348
14.6 Bounds on the Combined Strength of Pseudoshocks 351
14.7 Compactness and Consistency 354
14.8 Continuous Dependence on Initial Data 356
14.9 The Standard Riemann Semigroup 360
14.10 Uniqueness of Solutions 361
14.11 Structure of Solutions 366
14.12 Notes 366

Chapter XV. Compensated Compactness 371
15.1 The Young Measure 371
15.2 Compensated Compactness and the div-curl Lemma 373
15.3 Measure-Valued Solutions for Systems of Conservation Laws and Compensated Compactness 375
15.4 Scalar Conservation Laws 378
15.5 A Relaxation Scheme for Scalar Conservation Laws 379
15.6 Genuinely Nonlinear Systems of Two Conservation Laws 383
15.7 The System of Isentropic Elasticity 386
15.8 The System of Isentropic Gas Dynamics 390
15.9 Notes 392

Bibliography 397

Author Index 435

Subject Index 439