

# Contents

<b>1</b>	<b>Introduction</b> .....	1
<b>2</b>	<b>Quantisation of the Electromagnetic Field</b> .....	7
2.1	Field Quantisation .....	7
2.2	Fock or Number States .....	10
2.3	Coherent States .....	12
2.4	Squeezed States .....	15
2.5	Two-Photon Coherent States .....	18
2.6	Variance in the Electric Field .....	20
2.7	Multimode Squeezed States .....	22
2.8	Phase Properties of the Field .....	23
	Exercises .....	26
	References .....	26
	Further Reading .....	27
<b>3</b>	<b>Coherence Properties of the Electromagnetic Field</b> .....	29
3.1	Field-Correlation Functions .....	29
3.2	Properties of the Correlation Functions .....	31
3.3	Correlation Functions and Optical Coherence .....	32
3.4	First-Order Optical Coherence .....	34
3.5	Coherent Field .....	37
3.6	Photon Correlation Measurements .....	38
3.7	Quantum Mechanical Fields .....	41
3.7.1	Squeezed State .....	42
3.7.2	Squeezed Vacuum .....	44
3.8	Phase-Dependent Correlation Functions .....	44
3.9	Photon Counting Measurements .....	46
3.9.1	Classical Theory .....	46
3.9.2	Constant Intensity .....	48
3.9.3	Fluctuating Intensity–Short-Time Limit .....	48

3.10	Quantum Mechanical Photon Count Distribution	50
3.10.1	Coherent Light	51
3.10.2	Chaotic Light	51
3.10.3	Photo-Electron Current Fluctuations	52
	Exercises	54
	References	55
	Further Reading	55
<b>4</b>	<b>Representations of the Electromagnetic Field</b>	<b>57</b>
4.1	Expansion in Number States	57
4.2	Expansion in Coherent States	58
4.2.1	P Representation	58
4.2.2	Wigner's Phase-Space Density	62
4.2.3	Q Function	65
4.2.4	R Representation	67
4.2.5	Generalized P Representations	68
4.2.6	Positive P Representation	71
	Exercises	72
	References	72
<b>5</b>	<b>Quantum Phenomena in Simple Systems in Nonlinear Optics</b>	<b>73</b>
5.1	Single-Mode Quantum Statistics	73
5.1.1	Degenerate Parametric Amplifier	73
5.1.2	Photon Statistics	75
5.1.3	Wigner Function	76
5.2	Two-Mode Quantum Correlations	77
5.2.1	Non-degenerate Parametric Amplifier	77
5.2.2	Squeezing	80
5.2.3	Quadrature Correlations and the Einstein-Podolsky-Rosen Paradox	82
5.2.4	Wigner Function	83
5.2.5	Reduced Density Operator	84
5.3	Quantum Limits to Amplification	86
5.4	Amplitude Squeezed State with Poisson Photon Number Statistics	88
	Exercises	91
	References	91
<b>6</b>	<b>Stochastic Methods</b>	<b>93</b>
6.1	Master Equation	93
6.2	Equivalent c-Number Equations	99
6.2.1	Photon Number Representation	99
6.2.2	P Representation	100
6.2.3	Properties of Fokker-Planck Equations	102
6.2.4	Steady State Solutions - Potential Conditions	103
6.2.5	Time Dependent Solution	104

6.2.6	Q Representation	105
6.2.7	Wigner Function	107
6.2.8	Generalized P Representation	109
6.3	Stochastic Differential Equations	112
6.3.1	Use of the Positive P Representation	115
6.4	Linear Processes with Constant Diffusion	116
6.5	Two Time Correlation Functions in Quantum Markov Processes	117
6.5.1	Quantum Regression Theorem	118
6.6	Application to Systems with a P Representation	118
6.7	Stochastic Unravellings	119
6.7.1	Simulating Quantum Trajectories	123
	Exercises	124
	References	125
	Further Reading	125
<b>7</b>	<b>Input–Output Formulation of Optical Cavities</b>	<b>127</b>
7.1	Cavity Modes	127
7.2	Linear Systems	131
7.3	Two-Sided Cavity	132
7.4	Two Time Correlation Functions	133
7.5	Spectrum of Squeezing	135
7.6	Parametric Oscillator	136
7.7	Squeezing in the Total Field	138
7.8	Fokker–Planck Equation	138
	Exercises	141
	References	141
	Further Reading	141
<b>8</b>	<b>Generation and Applications of Squeezed Light</b>	<b>143</b>
8.1	Parametric Oscillation and Second Harmonic Generation	143
8.1.1	Semi-Classical Steady States and Stability Analysis	145
8.1.2	Parametric Oscillation	146
8.1.3	Second Harmonic Generation	146
8.1.4	Squeezing Spectrum	147
8.1.5	Parametric Oscillation	148
8.1.6	Experiments	149
8.2	Twin Beam Generation and Intensity Correlations	151
8.2.1	Second Harmonic Generation	156
8.2.2	Experiments	157
8.3	Applications of Squeezed Light	158
8.3.1	Interferometric Detection of Gravitational Radiation	158
8.3.2	Sub-Shot-Noise Phase Measurements	171
8.3.3	Quantum Information	173
	Exercises	174

References .....	174
Further Reading .....	175
<b>9 Nonlinear Quantum Dissipative Systems .....</b>	<b>177</b>
9.1 Optical Parametric Oscillator: Complex P Function .....	177
9.2 Optical Parametric Oscillator: Positive P Function .....	181
9.3 Quantum Tunnelling Time .....	186
9.4 Dispersive Optical Bistability .....	190
9.5 Comment on the Use of the Q and Wigner Representations .....	192
Exercises .....	192
9.A Appendix .....	193
9.A.1 Evaluation of Moments for the Complex P function for Parametric Oscillation (9.17) .....	193
9.A.2 Evaluation of the Moments for the Complex P Function for Optical Bistability (9.48) .....	194
References .....	195
Further Reading .....	195
<b>10 Interaction of Radiation with Atoms .....</b>	<b>197</b>
10.1 Quantization of the Many-Electron System .....	197
10.2 Interaction of a Single Two-Level Atom with a Single Mode Field ..	201
10.3 Spontaneous Emission from a Two-Level Atom .....	203
10.4 Phase Decay in a Two-Level System .....	204
10.5 Resonance Fluorescence .....	205
Exercises .....	210
References .....	210
Further Reading .....	211
<b>11 CQED .....</b>	<b>213</b>
11.1 Cavity QED .....	213
11.1.1 Vacuum Rabi Splitting .....	217
11.1.2 Single Photon Sources .....	218
11.1.3 Cavity QED with $N$ Atoms .....	221
11.2 Circuit QED .....	225
Exercises .....	227
References .....	228
Further Reading .....	229
<b>12 Quantum Theory of the Laser .....</b>	<b>231</b>
12.1 Master Equation .....	231
12.2 Photon Statistics .....	233
12.2.1 Spectrum of Intensity Fluctuations .....	234
12.3 Laser Linewidth .....	237
12.4 Regularly Pumped Laser .....	238
12.A Appendix: Derivation of the Single-Atom Increment .....	242

Exercises .....	245
References .....	245
<b>13 Bells Inequalities in Quantum Optics .....</b>	<b>247</b>
13.1 The Einstein–Podolsky–Rosen (EPR) Argument .....	247
13.2 Bell Inequalities and the Aspect Experiment .....	248
13.3 Violations of Bell’s Inequalities Using a Parametric Amplifier	
Source .....	254
13.4 One-Photon Interference .....	259
Exercises .....	264
References .....	264
<b>14 Quantum Nondemolition Measurements .....</b>	<b>267</b>
14.1 Concept of a QND Measurement .....	268
14.2 Back Action Evasion .....	270
14.3 Criteria for a QND Measurement .....	270
14.4 The Beam Splitter .....	273
14.5 Ideal Quadrature QND Measurements .....	276
14.6 Experimental Realisation .....	277
14.7 A Photon Number QND Scheme .....	279
Exercises .....	281
References .....	282
<b>15 Quantum Coherence and Measurement Theory .....</b>	<b>283</b>
15.1 Quantum Coherence .....	283
15.2 The Effect of Dissipation .....	288
15.2.1 Experimental Observation of Coherence Decay .....	291
15.3 Quantum Measurement Theory .....	293
15.3.1 General Measurement Theory .....	294
15.3.2 The Pointer Basis .....	296
15.4 Examples of Pointer Observables .....	299
15.5 Model of a Measurement .....	299
15.6 Conditional States and Quantum Trajectories .....	302
15.6.1 Homodyne Measurement of a Cavity Field .....	303
Exercises .....	305
References .....	306
<b>16 Quantum Information .....</b>	<b>307</b>
16.1 Introduction .....	307
16.1.1 The Qubit .....	308
16.1.2 Entanglement .....	310
16.2 Quantum Key Distribution .....	312
16.3 Quantum Teleportation .....	318
16.4 Quantum Computation .....	324
16.4.1 Linear Optical Quantum Gates .....	327
16.4.2 Single Photon Sources .....	336
Exercises .....	343

References .....	344
Further Reading .....	346
<b>17 Ion Traps .....</b>	<b>347</b>
17.1 Introduction .....	347
17.2 Trapping and Cooling .....	347
17.3 Novel Quantum States .....	353
17.4 Trapping Multiple Ions .....	356
17.5 Ion Trap Quantum Information Processing .....	359
Exercises .....	362
References .....	363
<b>18 Light Forces .....</b>	<b>365</b>
18.1 Radiative Forces in the Semiclassical Limit .....	366
18.2 Mean Force for a Two-Level Atom Initially at Rest .....	368
18.3 Friction Force for a Moving Atom .....	371
18.3.1 Laser Standing Wave—Doppler Cooling .....	372
18.4 Dressed State Description of the Dipole Force .....	374
18.5 Atomic Diffraction by a Standing Wave .....	377
18.6 Optical Stern–Gerlach Effect .....	381
18.7 Quantum Chaos .....	385
18.7.1 Dynamical Tunnelling .....	387
18.7.2 Dynamical Localisation .....	389
18.8 The Effect of Spontaneous Emission .....	390
References .....	394
Further Reading .....	395
<b>19 Bose-Einstein Condensation .....</b>	<b>397</b>
19.1 Hamiltonian: Binary Collision Model .....	398
19.2 Mean-Field Theory — Gross-Pitaevskii Equation .....	399
19.3 Single Mode Approximation .....	400
19.4 Quantum State of the Condensate .....	401
19.5 Quantum Phase Diffusion: Collapses and Revivals of the Condensate Phase .....	401
19.6 Interference of Two Bose–Einstein Condensates and Measurement-Induced Phase .....	405
19.6.1 Interference of Two Condensates Initially in Number States .....	405
19.7 Quantum Tunneling of a Two Component Condensate .....	409
19.7.1 Semiclassical Dynamics .....	411
19.7.2 Quantum Dynamics .....	414
19.8 Coherence Properties of Bose–Einstein Condensates .....	416
19.8.1 1st Order Coherence .....	416
19.8.2 Higher Order Coherence .....	417
Exercises .....	419
References .....	419
Further Reading .....	420
<b>Index .....</b>	<b>421</b>