Contents

Preface vii

1 Mathematical Models 1
1.1 Population Dynamics . . . 2
1.1.1 Notes and References . . . 6
1.2 Economics . . . 8
1.2.1 Notes and References . . . 12
1.3 Mechanics . . . 13
1.3.1 Translational Mechanical Systems . . . 13
1.3.2 Mechanical Systems with Rotational Elements . . . 18
1.3.3 The Variational Method . . . 27
1.3.4 Notes and References . . . 38
1.4 Electromagnetism and Electrical Systems . . . 39
1.4.1 Maxwell's Equations and the Elements of Electrical Circuits . . . 39
1.4.2 Electrical Networks . . . 50
1.4.3 Notes and References . . . 55
1.5 Digital Systems . . . 56
1.5.1 Combinational Switching Networks . . . 59
1.5.2 Sequential Switching Networks . . . 62
1.5.3 Notes and References . . . 68
1.6 Heat Transfer . . . 70
1.6.1 Notes and References . . . 72

2 Introduction to State Space Theory 73
2.1 Dynamical Systems . . . 74
2.1.1 The General Concept of a Dynamical System . . . 74
2.1.2 Differentiable Dynamical Systems . . . 83
2.1.3 System Properties . . . 88
2.1.4 Linearization . . . 92
2.1.5 Exercises . . . 94
2.1.6 Notes and References . . . 98
2.2 Linear Systems . . . 100
2.2.1 General Linear Systems . . . 100
2.2.2 Free Motions of Time Invariant Linear Differential Systems . . . 104
2.2.3 Free Motions of Time Invariant Linear Difference Systems . . . 113
2.2.4 Infinite Dimensional Systems . . . 115
2.2.5 Exercises . . . 121
2.2.6 Notes and References . . . 123
2.3 Linear Systems: Input Output Behaviour . . . 124

2.3.1 Input-Output Behaviour in Time Domain . . . 124
2.3.2 Transfer Functions . . . 138
2.3.3 Relationship Between Input–Output Operators and Transfer Matrices . . . 147
2.3.4 Exercises . . . 151
2.3.5 Notes and References . . . 153
2.4 Transformations and Interconnections . . . 154
2.4.1 Morphisms and Standard Constructions . . . 154
2.4.2 Composite Systems . . . 160
2.4.3 Exercises . . . 166
2.4.4 Notes and References . . . 167
2.5 Sampling and Approximation . . . 168
2.5.1 A/D- and D/A-Conversion of Signals . . . 169
2.5.2 The Sampling Theorem . . . 171
2.5.3 Sampling Continuous Time Systems . . . 175
2.5.4 Approximation of Continuous Systems by Discrete Systems . . . 177
2.5.5 Exercises . . . 189
2.5.6 Notes and References . . . 192

3 Stability Theory **193**
3.1 General Definitions . . . 194
3.1.1 Local Flows . . . 195
3.1.2 Stability Definitions . . . 199
3.1.3 Limit Sets . . . 202
3.1.4 Recurrence . . . 206
3.1.5 Attractors . . . 211
3.1.6 Exercises . . . 213
3.1.7 Notes and References . . . 215
3.2 Liapunov's Direct Method . . . 217
3.2.1 General Definitions and Results . . . 217
3.2.2 Time–Varying Finite Dimensional Systems . . . 229
3.2.3 Time–Invariant Systems . . . 235
3.2.4 Exercises . . . 248
3.2.5 Notes and References . . . 251
3.3 Linearization and Stability . . . 253
3.3.1 Stability Criteria for Time-Varying Linear Systems . . . 254
3.3.2 Time–Invariant Systems: Spectral Stability Criteria . . . 263
3.3.3 Numerical Stability of Discretization Methods . . . 268
3.3.4 Liapunov Functions for Time-Varying Linear Systems . . . 272
3.3.5 Liapunov Functions for Time-Invariant Linear Systems . . . 282
3.3.6 Exercises . . . 291
3.3.7 Notes and References . . . 295
3.4 Stability Criteria for Polynomials . . . 296
3.4.1 Stability Criteria and the Argument Principle . . . 297
3.4.2 Characterization of Stability via the Cauchy Index . . . 308
3.4.3 Hermite Forms and Bézoutiants . . . 313
3.4.4 Hankel Matrices and Rational Functions . . . 320
3.4.5 Applications to Stability . . . 334
3.4.6 Schur Polynomials . . . 340

3.4.7 Algebraic Stability Domains and Linear Matrix Equations 357
3.4.8 Exercises 361
3.4.9 Notes and References 366

4 Perturbation Theory **369**
4.1 Perturbation of Polynomials 369
4.1.1 Dependence of the Roots on the Coefficient Vector 370
4.1.2 Polynomials with Holomorphic Coefficients 376
4.1.3 The Sets of Hurwitz and Schur Polynomials 384
4.1.4 Kharitonov's Theorem 389
4.1.5 Exercises 393
4.1.6 Notes and References 396
4.2 Perturbation of Matrices 398
4.2.1 Continuity and Analyticity of Eigenvalues 398
4.2.2 Estimates for Eigenvalues and Growth Rates 404
4.2.3 Smoothness of Eigenprojections and Eigenvectors 409
4.2.4 Exercises 426
4.2.5 Notes and References 429
4.3 The Singular Value Decomposition 431
4.3.1 Singular Values and Singular Vectors 431
4.3.2 Singular Value Decomposition 435
4.3.3 Matrices Depending on a Real Parameter 439
4.3.4 Relations between Eigenvalues and Singular Values 444
4.3.5 Exercises 446
4.3.6 Notes and References 448
4.4 Structured Perturbations 449
4.4.1 Elements of μ-Analysis 449
4.4.2 μ-Values for Real Full-Block Perturbations 465
4.4.3 Exercises 480
4.4.4 Notes and References 481
4.5 Computational Aspects 484
4.5.1 Condition Numbers 485
4.5.2 Matrix Transformations 492
4.5.3 Algorithms 501
4.5.4 Exercises 513
4.5.5 Notes and References 515

5 Uncertain Systems **517**
5.1 Models of Uncertainty and Tools for their Analysis 520
5.1.1 General Definitions and Basic Properties 520
5.1.2 Perturbation Structures 530
5.1.3 Exercises 540
5.1.4 Notes and References 542
5.2 Spectral Value Sets 544
5.2.1 General Definitions and Results 544
5.2.2 Complex Full-Block Perturbations 556
5.2.3 Real Full-Block Perturbations 561
5.2.4 The Unstructured Case (Pseudospectra) 569
5.2.5 Exercises 580

5.2.6 Notes and References . . . 583
5.3 Stability Radii . . . 585
5.3.1 General Definitions and Results . . . 586
5.3.2 Complex Full-Block Perturbations . . . 591
5.3.3 Real Full-Block Perturbations . . . 596
5.3.4 Hamiltonian Characterization of the Complex Stability Radius . . . 602
5.3.5 The Unstructured Case . . . 609
5.3.6 Dependence on System Data . . . 614
5.3.7 Stability Radii and the Cayley Transformation . . . 617
5.3.8 Exercises . . . 621
5.3.9 Notes and References . . . 624
5.4 Root Sets and Stability Radii of Polynomials . . . 625
5.4.1 General Formulas . . . 625
5.4.2 Complex Perturbation Structures . . . 633
5.4.3 Real Perturbation Structures . . . 637
5.4.4 Exercises . . . 644
5.4.5 Notes and References . . . 646
5.5 Transient Behaviour . . . 648
5.5.1 Transient Bounds and Initial Growth Rate . . . 648
5.5.2 Contractions and Estimates of the Transient Bound . . . 658
5.5.3 Spectral Value Sets and Transient Behaviour . . . 669
5.5.4 Robustness of (M, β)-Stability . . . 675
5.5.5 Exercises . . . 680
5.5.6 Notes and References . . . 684
5.6 More General Perturbation Classes . . . 686
5.6.1 The Perturbation Classes . . . 687
5.6.2 Stability Radii . . . 696
5.6.3 The Aizerman Conjecture . . . 701
5.6.4 Exercises . . . 709
5.6.5 Notes and References . . . 711

Appendix **715**
A.1 Linear Algebra . . . 715
A.1.1 Norms of Vectors and Matrices . . . 715
A.1.2 Spectra and Determinants . . . 719
A.1.3 Real Representation of Complex Matrices . . . 720
A.1.4 Direct Sums and Kronecker Products . . . 720
A.1.5 Hermitian Matrices . . . 722
A.2 Complex Analysis . . . 724
A.2.1 Topological Preliminaries . . . 724
A.2.2 Path Integrals . . . 725
A.2.3 Holomorphic Functions . . . 727
A.2.4 Isolated Singularities . . . 729
A.2.5 Analytic Continuation . . . 732
A.2.6 Maximum Principle and Subharmonic Functions . . . 733
A.3 Convolutions and Transforms . . . 735
A.3.1 Sequences: Convolution and **z**-Transforms . . . 735
A.3.2 Lebesgue Spaces, Convolution of Functions, Laplace Transforms . . . 739
A.3.3 Fourier Series and Fourier Transforms . . . 744

A.3.4 Hardy Spaces 750
A.4 Linear Operators and Linear Forms 753
A.4.1 Summability and Generalized Fourier Series 753
A.4.2 Linear Operators on Banach Spaces 754
A.4.3 Linear Operators on Hilbert Spaces 757
A.4.4 Spectral Theory 759

References **763**

Glossary **789**

Index **795**