

# Contents

<b>1</b>	<b>Overview</b> .....	1
1.1	Introduction .....	1
1.2	Microeconomic Foundations .....	2
1.3	Efficiency Measurement .....	6
1.4	Productivity and Performance Measurement .....	8
1.5	Engineering Models of Technology .....	10
1.6	Mathematical Appendix .....	13
1.7	A Word of Advice .....	14

---

## Part I Microeconomic Foundations

---

<b>2</b>	<b>Production Functions</b> .....	19
2.1	Parametric Forms .....	19
2.2	Rate of Technical Substitution .....	21
2.3	Elasticity .....	24
2.4	Elasticity of Output, Scale and Returns to Scale .....	25
2.5	Elasticity of Substitution .....	26
2.6	Homothetic Production Functions .....	28
2.7	Exercises .....	30
2.8	Bibliographical Notes .....	31
2.9	Solutions to Exercises .....	32
<b>3</b>	<b>Formal Description of Technology</b> .....	35
3.1	Primitive Elements .....	35
3.2	Input and Output Disposability .....	37
3.3	Efficient Frontiers .....	38
3.4	Axioms for a Well-Behaved Technology .....	38
3.5	Single-Output Technologies .....	39
3.6	Extrapolation of Technology .....	41
3.6.1	Convexity .....	41

3.6.2	Disposability .....	42
3.6.3	Constant Returns-to-Scale .....	43
3.6.4	Example .....	45
3.7	Exercises .....	47
3.8	Bibliographical Notes .....	48
3.9	Solutions to Exercises .....	49
<b>4</b>	<b>Nonparametric Models of Technology</b> .....	<b>53</b>
4.1	Simple Leontief or Fixed-Coefficients Technology .....	53
4.2	General Leontief Technology .....	55
4.2.1	Production Function .....	56
4.2.2	Properties .....	56
4.2.3	Graphical Construction .....	58
4.3	Nonparametric Constructions .....	60
4.3.1	The Hanoch-Rothschild Model of Technology .....	60
4.3.2	Data Envelopment Analysis Models of Technology .....	61
4.3.3	Graphical Constructions .....	63
4.4	Exercises .....	66
4.5	Bibliographical Notes .....	67
4.6	Solutions to Exercises .....	68
<b>5</b>	<b>Cost Function</b> .....	<b>71</b>
5.1	Definition .....	71
5.2	Properties .....	71
5.2.1	Geometry .....	71
5.2.2	Homogeneity .....	73
5.2.3	Concavity .....	73
5.3	Example: Cobb-Douglas Technology .....	73
5.4	Sensitivity Analysis .....	76
5.4.1	Sensitivity to Output .....	76
5.4.2	Sensitivity to Price: Shephard's Lemma .....	77
5.5	Nonparametric Estimation .....	78
5.5.1	Leontief Technologies .....	78
5.5.2	<i>HR</i> Technology .....	79
5.5.3	<i>CRS</i> and <i>VRS</i> Technologies .....	79
5.6	Reconstructing the Technology .....	80
5.6.1	Outer Approximation of Technology .....	82
5.6.2	Cost and Production .....	83
5.7	Homothetic Technologies .....	84
5.8	Appendix .....	85
5.9	Exercises .....	86
5.10	Bibliographical Notes .....	89
5.11	Solutions to Exercises .....	90

<b>6</b>	<b>Indirect Production Function</b> .....	97
6.1	Definition .....	97
6.2	Properties .....	98
6.3	Duality between the Cost and Indirect Production Functions .	99
6.4	Reconstructing the Technology .....	100
6.5	Revealed Preference .....	101
6.6	Nonparametric Estimation .....	102
6.7	Exercises .....	103
6.8	Bibliographical Notes .....	104
6.9	Solutions to Exercises .....	105
<b>7</b>	<b>Distance Functions</b> .....	109
7.1	Definition .....	109
	7.1.1 Input Distance Function .....	109
	7.1.2 Output Distance Function .....	110
7.2	Properties .....	111
7.3	Efficiency and Cost .....	112
7.4	Reconstructing the Input Distance Function from the Cost Function .....	114
7.5	Application to Homothetic Technologies .....	117
7.6	Appendix .....	118
7.7	Exercises .....	120
7.8	Bibliographical Notes .....	120
7.9	Solutions to Exercises .....	121
<b>8</b>	<b>Nonconvex Models of Technology</b> .....	125
8.1	Resource Allocation .....	125
	8.1.1 Aggregate Production Function .....	126
	8.1.2 Counter-Example to Quasiconcavity .....	127
8.2	Producer Budgeting .....	129
	8.2.1 Multi-Dimensional Indirect Production Function .....	129
	8.2.2 Counter-Example to Quasiconvexity .....	129
8.3	Data Envelopment Analysis with Lower Bounds .....	130
	8.3.1 Fixed-Charge Technology .....	130
	8.3.2 Nonconvex Geometry of the Fixed-Charge Technology .	132
	8.3.3 The Low Intensity Phenomenon .....	133
8.4	Projective-Convexity .....	135
	8.4.1 Definitions and characterizations .....	136
	8.4.2 Separation Properties .....	139
	8.4.3 Dual Characterization .....	141
8.5	Exercises .....	143
8.6	Bibliographical Notes .....	143
8.7	Solutions to Exercises .....	144

---

**Part II Efficiency Measurement**

---

<b>9</b>	<b>Efficiency Analysis</b> .....	149
9.1	Input and Output Efficiency .....	149
9.2	Scale Efficiency .....	151
9.3	Cost Efficiency .....	152
9.4	Joint Input-Output Efficiency .....	153
9.5	Computing Input Efficiency .....	154
9.5.1	CRS Technology .....	154
9.5.2	VRS Technology .....	156
9.5.3	HR Technology .....	156
9.6	Computing Output Efficiency .....	157
9.7	Computing Cost Efficiency .....	158
9.8	Computing Joint Input-Output Efficiency .....	158
9.9	Exercises .....	159
9.10	Bibliographical Notes .....	161
9.11	Solutions to Exercises .....	162
<b>10</b>	<b>The Two-Dimensional Projection</b> .....	167
10.1	Definition .....	167
10.2	Characterizations .....	168
10.3	Computing Efficiency .....	171
10.4	Scale Characterizations .....	172
10.5	Example .....	172
10.6	Extensions .....	175
10.7	Pivoting Algorithm .....	176
10.7.1	Vertices and the Simplex Tableau .....	177
10.7.2	Pivot Operation .....	178
10.7.3	Phase I .....	180
10.7.4	Phase II .....	181
10.8	Exercises .....	183
10.9	Bibliographical Notes .....	185
10.10	Solutions to Exercises .....	186
<b>11</b>	<b>Multi-Stage Efficiency Analysis</b> .....	191
11.1	A Representative Multi-Stage System .....	192
11.2	Description of Multi-Stage Technology .....	193
11.2.1	Classical Models of Technology .....	193
11.2.2	Expanded Model of Technology .....	194
11.2.3	Expanded Subsystem Technology Sets .....	196
11.3	Pareto efficient Frontiers .....	197
11.4	Aggregate Efficiency .....	199
11.4.1	Measures of Aggregate Input Efficiency .....	199
11.4.2	Derived Measure of Aggregate Efficiency .....	200

11.4.3	Computational Results .....	201
11.5	A Consistent Pricing Principle .....	203
11.6	Extensions .....	205
11.7	Bibliographical Notes .....	205
<b>12</b>	<b>Efficiency Analysis of Warehouse and Distribution Operations .....</b>	<b>207</b>
12.1	Business Environment .....	207
12.2	Description of Technology .....	208
12.2.1	Input Categories .....	208
12.2.2	Output Categories .....	209
12.2.3	Caveats .....	211
12.3	Measuring Operating Efficiency .....	211
12.4	Empirical Results .....	214
12.5	Current Assessment .....	215
12.6	Data and Results .....	216
12.7	Exercises .....	220
12.8	Bibliographical Notes .....	220

---

**Part III Productivity and Performance Measurement**

---

<b>13</b>	<b>Index Numbers .....</b>	<b>223</b>
13.1	Motivating Example .....	223
13.2	Price Indexes .....	227
13.2.1	Konus Price Index .....	227
13.2.2	Laspeyres and Paasche Price Indexes .....	228
13.3	Fisher and Tornqvist Price Indexes .....	230
13.3.1	Fisher Ideal Price Index .....	230
13.3.2	Tornqvist Price Index .....	231
13.4	Implicit Quantity Indexes .....	233
13.5	Quantity Indexes .....	233
13.6	Implicit Price Indexes .....	234
13.7	Exercises .....	235
13.8	Bibliographical Notes .....	237
13.9	Solutions to Exercises .....	238
<b>14</b>	<b>Productivity Measurement .....</b>	<b>241</b>
14.1	Growth Rates .....	241
14.2	Growth Accounting Approach .....	243
14.3	Multi-Output Productivity Measurement .....	245
14.4	Nonparametric Approach .....	246
14.4.1	Input Productivity Change .....	246
14.4.2	Output Productivity Change .....	248
14.5	Exercises .....	250

14.6	Bibliographical Notes . . . . .	252
14.7	Solutions to Exercises . . . . .	253
<b>15</b>	<b>Performance Measurement . . . . .</b>	<b>257</b>
15.1	A Manufacturing Example . . . . .	257
15.2	Performance Indexes . . . . .	259
15.3	Productivity Assessment . . . . .	261
15.4	Performance Ratios . . . . .	262
15.4.1	Profitability Ratio . . . . .	263
15.4.2	Productivity Ratio . . . . .	264
15.4.3	Price Recovery Ratio . . . . .	264
15.5	Distribution of Net Gain . . . . .	265
15.5.1	Net Gain . . . . .	266
15.5.2	Net Gain Due to Productivity . . . . .	267
15.5.3	Net Gain Due to Price Recovery . . . . .	267
15.6	Exercises . . . . .	268
15.7	Bibliographical Notes . . . . .	268
15.8	Solutions to Exercises . . . . .	269
<b>16</b>	<b>Economic Analysis . . . . .</b>	<b>271</b>
16.1	Market Structure and Equilibrium . . . . .	271
16.2	Competitive Market Structure . . . . .	273
16.2.1	Consumers . . . . .	273
16.2.2	Producers . . . . .	274
16.2.3	Equilibrium . . . . .	274
16.2.4	Comparative Statics . . . . .	276
16.3	Monopolistic Competitive Market Structure . . . . .	277
16.4	Social Planner's Perspective . . . . .	278
16.5	Oligopoly Market Structure . . . . .	279
16.5.1	Profit Maximization Formulation . . . . .	279
16.5.2	Equilibrium . . . . .	280
16.5.3	Algorithm to Compute the Equilibrium . . . . .	282
16.5.4	Comparison to Competitive and Monopolistic Competitive Market Structures . . . . .	283
16.6	Productivity Analysis . . . . .	284
16.6.1	Analysis of a Productivity Laggard . . . . .	284
16.6.2	Analysis of a Productivity Leader . . . . .	284
16.7	Exercises . . . . .	284
16.8	Bibliographical Notes . . . . .	287
16.9	Solutions to Exercises . . . . .	288

---

**Part IV Engineering Models of Technology**

---

<b>17</b>	<b>Index-Based Dynamic Production Functions</b>	295
17.1	A Motivating Example	295
17.2	Input-Output Domain	297
17.2.1	Event-Based Flows	298
17.2.2	Rate-Based Flows	298
17.3	Instantaneous Processes	298
17.4	Index-Based Processes	299
17.4.1	Definition	299
17.4.2	Fixed Proportions, Instantaneous Model	300
17.4.3	Fixed Proportions, Constant Lead Time Models	300
17.5	Exercises	304
17.6	Bibliographical Notes	305
17.7	Solutions to Exercises	306
<b>18</b>	<b>Distribution-Based Dynamic Production Functions</b>	309
18.1	Description	309
18.1.1	Overview	309
18.1.2	Definition	310
18.1.3	Lead Time Density	311
18.1.4	Technical Remarks	312
18.2	Constant Lead Time Processes	313
18.2.1	Description	313
18.2.2	Integer Lead Times	314
18.2.3	Noninteger Lead Times	315
18.2.4	Non-Integer Lead Times with Unequal Length Periods	318
18.3	Time-Dependent Lead Time Processes	320
18.3.1	Description	320
18.3.2	First-In, First-Out Example	321
18.3.3	Leapfrog Example	322
18.4	Continuous Lead Time Processes	324
18.4.1	Description	324
18.4.2	Examples	326
18.5	Exercises	329
18.6	Solutions to Exercises	331
<b>19</b>	<b>Dynamic Production Function Approximations</b>	337
19.1	Load-Dependent Processes	337
19.1.1	Formulation	338
19.1.2	Example	339
19.1.3	Linear Approximation	340
19.1.4	Load-Dependent, Linear Approximation	346

XVIII Contents

19.2	Two-Point Boundary Approximation	348
19.2.1	Relative Area Ratio	349
19.2.2	Linear Approximation	350
19.2.3	Example	351
19.2.4	Extensions	353
19.3	Application to Project-Oriented Production Systems	356
19.3.1	Description	356
19.3.2	Detailed Activities	357
19.3.3	Aggregate Activities	358
19.3.4	Aggregate Dynamic Production Function	360
19.4	Aggregation of Dynamic Production Functions	361
19.4.1	Serial Aggregation	361
19.4.2	Parallel Aggregation	362
19.5	Estimation via Dynamic Activity Analysis	362
19.5.1	Basic Model	362
19.5.2	Extensions	363
19.6	Exercises	364
19.7	Bibliographical Notes	365
19.8	Solutions to Exercises	366
<b>20</b>	<b>A Stochastic Input-Output Model</b>	<b>373</b>
20.1	Input-Output Model with Single Inputs	373
20.2	Input-Output Model with Batch Input	375
20.2.1	Simultaneous Batch Case	376
20.2.2	Independent Batch Case	377
20.3	Confidence intervals	378
20.3.1	Without Batch Input	378
20.3.2	With Batch Input	379
20.3.3	Linear Approximation	382
20.4	Exercises	383
20.5	Bibliographical Notes	385
20.6	Solutions to Exercises	386
<b>21</b>	<b>Multi-Stage, Dynamic Models of Technology</b>	<b>391</b>
21.1	Basic Model	392
21.1.1	Primitives	392
21.1.2	Material Balance and Service Capacity Constraints	393
21.2	Index-Based Models	394
21.2.1	Instantaneous Processes	394
21.2.2	Constant Lead Time Processes	395
21.2.3	Multi-Event, Constant Lead Time Processes	396
21.2.4	Continuous Lead Time Based Processes	397
21.2.5	Initial Conditions	398
21.3	Computational Models	401



- 21.4 A Manufacturing Example . . . . . 404
  - 21.4.1 Production Process Description . . . . . 404
  - 21.4.2 Formulation . . . . . 404
  - 21.4.3 Extensions . . . . . 407
- 21.5 Assembly with Rework Example . . . . . 408
  - 21.5.1 Production Process Description . . . . . 408
  - 21.5.2 Formulation . . . . . 409
  - 21.5.3 Extensions . . . . . 411
- 21.6 Extensions to the Basic Model . . . . . 411
  - 21.6.1 Material Balance Constraints . . . . . 411
  - 21.6.2 Transfers of Product or Materials . . . . . 412
  - 21.6.3 Activity Constraints . . . . . 412
  - 21.6.4 Service Output . . . . . 413
  - 21.6.5 Alternate Production Processes . . . . . 413
  - 21.6.6 Load-Dependent, Multi-Product, Single-Stage Model . . 414
- 21.7 Efficiency and Productivity Measurement . . . . . 417
  - 21.7.1 Input and Output Efficiency . . . . . 417
  - 21.7.2 Cost and Allocative Efficiency . . . . . 417
  - 21.7.3 Productivity Assessment . . . . . 418
  - 21.7.4 Computation . . . . . 418
- 21.8 Bibliographical Notes . . . . . 418
  
- 22 Optimizing Labor Resources Within a Warehouse . . . . . 421**
  - 22.1 Introduction . . . . . 421
  - 22.2 System Description . . . . . 422
    - 22.2.1 Business Environment . . . . . 422
    - 22.2.2 Material Flow . . . . . 423
    - 22.2.3 Workforce Schedule . . . . . 423
    - 22.2.4 Sources of Inefficiency . . . . . 423
  - 22.3 An Optimization Model . . . . . 425
    - 22.3.1 Parameters . . . . . 425
    - 22.3.2 Decision Variables . . . . . 426
    - 22.3.3 Constraints . . . . . 427
    - 22.3.4 Objective Function . . . . . 428
  - 22.4 Implementation . . . . . 429
    - 22.4.1 Computational Issues . . . . . 429
    - 22.4.2 Using the Prototype Model: A Case Study . . . . . 430
    - 22.4.3 Benefits and Other Applications . . . . . 430
  - 22.5 Bibliographical Notes . . . . . 431

---

**Part V Mathematical Appendix**


---

<b>A</b>	<b>Notation and Mathematical Preliminaries</b>	435
A.1	Logical Statements	435
A.2	Sets	435
A.3	Vectors	438
A.4	Correspondences	439
A.5	Functions	440
A.6	Matrices	442
A.7	Differentiability	444
<b>B</b>	<b>Real Analysis</b>	449
B.1	Linear Spaces	449
B.1.1	Definition	449
B.1.2	Examples	450
B.2	Linear Independence and Dimension	451
B.3	Normed Linear Spaces	451
B.3.1	Definition	451
B.3.2	Examples	452
B.4	Metric Spaces	453
B.4.1	Definition	453
B.4.2	Open and Closed Sets	454
B.4.3	Closure and Boundary	454
B.4.4	Convergence and Limits	456
B.4.5	Completeness	456
B.4.6	Compactness	457
B.4.7	Continuity	458
B.4.8	Connectedness	460
B.5	Bibliographical Notes	460
<b>C</b>	<b>Convex Sets</b>	461
C.1	Definition and Examples	461
C.2	Convexification	462
C.3	Separation of a Convex Set and a Point	463
C.3.1	Strict Separation	463
C.3.2	Supporting Hyperplanes	464
C.3.3	Polar Cones	464
C.4	Polyhedra	465
C.4.1	Definition and Examples	465
C.4.2	Extreme Points and Directions	466
C.4.3	Characterization of Extreme Points and Directions	467
C.4.4	Representation Theorem for Polyhedra	470
C.5	Application to Linear Programming	471
C.6	Bibliographical Notes	472

**D Concave, Convex Functions and Generalizations** . . . . . 473

    D.1 Definitions . . . . . 473

    D.2 Quasiconcavity and Quasiconvexity . . . . . 474

    D.3 Differential Characterizations . . . . . 476

**E Optimality Conditions** . . . . . 479

    E.1 Unconstrained Problems . . . . . 479

    E.2 Problems with Inequality Constraints . . . . . 480

    E.3 Lagrangian Duality . . . . . 482

    E.4 Application of Duality to Economic Lot Sizes . . . . . 486

    E.5 Application of Duality to Linear Programming . . . . . 487

    E.6 Bibliographical Notes . . . . . 489

**F Envelope Theorem** . . . . . 491

    F.1 Statement and Proof . . . . . 491

    F.2 Application to Sensitivity Analysis of Cost . . . . . 493

    F.3 A Monopoly Pricing Example . . . . . 493

    F.4 Bibliographical Notes . . . . . 494

**G Correspondence Theory** . . . . . 495

    G.1 Core Concepts . . . . . 495

    G.2 Characterization by Sequences . . . . . 498

    G.3 Bibliographical Notes . . . . . 499

**H Theorem of the Maximum** . . . . . 501

    H.1 Application to the Indirect Production Function . . . . . 502

    H.2 Application to the Cost Function . . . . . 503

    H.3 Bibliographical Notes . . . . . 505

**I Probability Basics** . . . . . 507

    I.1 Binomial Random Variables . . . . . 507

    I.2 Poisson Random Variables . . . . . 507

    I.3 Poisson Processes . . . . . 508

    I.4 Moment Generating Functions . . . . . 509

    I.5 Conditional Expectation and Variance . . . . . 509

    I.6 Bibliographical Notes . . . . . 510

**References** . . . . . 511

**Index** . . . . . 517