

---

# Contents

<b>Preface</b> .....	v
<b>Introduction</b> .....	1

---

## Part I *D*-Modules and Perverse Sheaves

---

<b>1 Preliminary Notions</b> .....	15
1.1 Differential operators .....	15
1.2 <i>D</i> -modules—warming up .....	17
1.3 Inverse and direct images I .....	21
1.4 Some categories of <i>D</i> -modules .....	25
1.5 Inverse images and direct images II .....	31
1.6 Kashiwara’s equivalence .....	48
1.7 A base change theorem for direct images .....	53
<b>2 Coherent <i>D</i>-Modules</b> .....	57
2.1 Good filtrations .....	57
2.2 Characteristic varieties (singular supports) .....	59
2.3 Dimensions of characteristic varieties .....	62
2.4 Inverse images in the non-characteristic case .....	64
2.5 Proper direct images .....	69
2.6 Duality functors .....	70
2.7 Relations among functors .....	76
2.7.1 Duality functors and inverse images .....	76
2.7.2 Duality functors and direct images .....	77
<b>3 Holonomic <i>D</i>-Modules</b> .....	81
3.1 Basic results .....	81
3.2 Functors for holonomic <i>D</i> -modules .....	83
3.2.1 Stability of holonomicity .....	83

3.2.2	Holonomicity of modules over Weyl algebras .....	86
3.2.3	Adjunction formulas .....	91
3.3	Finiteness property .....	93
3.4	Minimal extensions .....	95
<b>4</b>	<b>Analytic <math>D</math>-Modules and the de Rham Functor</b> .....	<b>99</b>
4.1	Analytic $D$ -modules .....	99
4.2	Solution complexes and de Rham functors .....	103
4.3	Cauchy–Kowalevski–Kashiwara theorem .....	105
4.4	Cauchy problems and micro-supports .....	107
4.5	Constructible sheaves .....	110
4.6	Kashiwara’s constructibility theorem .....	113
4.7	Analytic $D$ -modules associated to algebraic $D$ -modules .....	119
<b>5</b>	<b>Theory of Meromorphic Connections</b> .....	<b>127</b>
5.1	Meromorphic connections in the one-dimensional case .....	127
5.1.1	Systems of ODEs and meromorphic connections .....	127
5.1.2	Meromorphic connections with regular singularities .....	130
5.1.3	Regularity of $D$ -modules on algebraic curves .....	135
5.2	Regular meromorphic connections on complex manifolds .....	139
5.2.1	Meromorphic connections in higher dimensions .....	139
5.2.2	Meromorphic connections with logarithmic poles .....	143
5.2.3	Deligne’s Riemann–Hilbert correspondence .....	147
5.3	Regular integrable connections on algebraic varieties .....	153
<b>6</b>	<b>Regular Holonomic <math>D</math>-Modules</b> .....	<b>161</b>
6.1	Definition and main theorems .....	161
6.2	Proof of main theorems .....	163
<b>7</b>	<b>Riemann–Hilbert Correspondence</b> .....	<b>171</b>
7.1	Commutativity with de Rham functors .....	171
7.2	Riemann–Hilbert correspondence .....	174
7.3	Comparison theorem .....	178
<b>8</b>	<b>Perverse Sheaves</b> .....	<b>181</b>
8.1	Theory of perverse sheaves .....	181
8.1.1	$t$ -structures .....	181
8.1.2	Perverse sheaves .....	191
8.2	Intersection cohomology theory .....	202
8.2.1	Introduction .....	202
8.2.2	Minimal extensions of perverse sheaves .....	203
8.3	Hodge modules .....	217
8.3.1	Motivation .....	217
8.3.2	Hodge structures and their variations .....	217
8.3.3	Hodge modules .....	220

---

**Part II Representation Theory**


---

<b>9</b>	<b>Algebraic Groups and Lie Algebras</b> . . . . .	229
9.1	Lie algebras and their enveloping algebras . . . . .	229
9.2	Semisimple Lie algebras (1) . . . . .	231
9.3	Root systems . . . . .	233
9.4	Semisimple Lie algebras (2) . . . . .	236
9.5	Finite-dimensional representations of semisimple Lie algebras . . . . .	239
9.6	Algebraic groups and their Lie algebras . . . . .	241
9.7	Semisimple algebraic groups . . . . .	243
9.8	Representations of semisimple algebraic groups . . . . .	248
9.9	Flag manifolds . . . . .	250
9.10	Equivariant vector bundles . . . . .	253
9.11	The Borel–Weil–Bott theorem . . . . .	255
<b>10</b>	<b>Conjugacy Classes of Semisimple Lie Algebras</b> . . . . .	259
10.1	The theory of invariant polynomials . . . . .	259
10.2	Classification of conjugacy classes . . . . .	263
10.3	Geometry of conjugacy classes . . . . .	266
<b>11</b>	<b>Representations of Lie Algebras and <math>D</math>-Modules</b> . . . . .	271
11.1	Universal enveloping algebras and differential operators . . . . .	271
11.2	Rings of twisted differential operators on flag varieties . . . . .	272
11.3	Proof of Theorem 11.2.2 . . . . .	276
11.4	Proof of Theorems 11.2.3 and 11.2.4 . . . . .	280
11.5	Equivariant representations and equivariant $D$ -modules . . . . .	283
11.6	Classification of equivariant $D$ -modules . . . . .	286
<b>12</b>	<b>Character Formula of Highest Weight Modules</b> . . . . .	289
12.1	Highest weight modules . . . . .	289
12.2	Kazhdan–Lusztig conjecture . . . . .	295
12.3	$D$ -modules associated to highest weight modules . . . . .	299
<b>13</b>	<b>Hecke Algebras and Hodge Modules</b> . . . . .	305
13.1	Weyl groups and $D$ -modules . . . . .	305
13.2	Hecke algebras and Hodge modules . . . . .	310
<b>A</b>	<b>Algebraic Varieties</b> . . . . .	321
A.1	Basic definitions . . . . .	321
A.2	Affine varieties . . . . .	323
A.3	Algebraic varieties . . . . .	325
A.4	Quasi-coherent sheaves . . . . .	326
A.5	Smoothness, dimensions and local coordinate systems . . . . .	328

<b>B</b>	<b>Derived Categories and Derived Functors</b> .....	331
	B.1 Motivation .....	331
	B.2 Categories of complexes .....	333
	B.3 Homotopy categories .....	335
	B.4 Derived categories .....	339
	B.5 Derived functors .....	343
	B.6 Bifunctors in derived categories .....	347
<b>C</b>	<b>Sheaves and Functors in Derived Categories</b> .....	351
	C.1 Sheaves and functors .....	351
	C.2 Functors in derived categories of sheaves .....	355
	C.3 Non-characteristic deformation lemma .....	361
<b>D</b>	<b>Filtered Rings</b> .....	365
	D.1 Good filtration .....	365
	D.2 Global dimensions .....	367
	D.3 Singular supports .....	370
	D.4 Duality .....	372
	D.5 Codimension filtration .....	375
<b>E</b>	<b>Symplectic Geometry</b> .....	379
	E.1 Symplectic vector spaces .....	379
	E.2 Symplectic structures on cotangent bundles .....	380
	E.3 Lagrangian subsets of cotangent bundles .....	381
	<b>References</b> .....	387
	<b>List of Notation</b> .....	397
	<b>Index</b> .....	403