
Contents

0	Meta Contents	iii
	Preface	v
	Contents	ix
	Tables, Figures, Theorems,	xv
	Notation	xvii
1	A Short Tour Through the Book	1
1.1	Introduction	2
1.2	Simplicity & Uncertainty	3
1.2.1	Introduction	3
1.2.2	Algorithmic Information Theory	4
1.2.3	Uncertainty & Probabilities	5
1.2.4	Algorithmic Probability & Universal Induction	6
1.2.5	Generalized Universal (Semi)Measures	7
1.3	Universal Sequence Prediction	7
1.3.1	Setup & Convergence	8
1.3.2	Loss Bounds	8
1.3.3	Optimality Properties	9
1.3.4	Miscellaneous	10
1.4	Rational Agents in Known Probabilistic Environments	11
1.4.1	The Agent Model	11
1.4.2	Value Functions & Optimal Policies	11
1.4.3	Sequential Decision Theory & Reinforcement Learning ..	12
1.5	The Universal Algorithmic Agent AIXI	13
1.5.1	The Universal AIXI Model	13
1.5.2	On the Optimality of AIXI	14
1.5.3	Value-Related Optimality Results	15
1.5.4	Markov Decision Processes	17
1.5.5	The Choice of the Horizon	18
1.6	Important Environmental Classes	18
1.6.1	Introduction	18
1.6.2	Sequence Prediction (SP)	19
1.6.3	Strategic Games (SG)	19
1.6.4	Function Minimization (FM)	19
1.6.5	Supervised Learning from Examples (EX)	19
1.6.6	Other Aspects of Intelligence	20
1.7	Computational Aspects	20

1.7.1	The Fastest & Shortest Algorithm for All Problems	20
1.7.2	Time-Bounded AIXI Model	22
1.8	Discussion	24
1.9	History & References	26
2	Simplicity & Uncertainty	29
2.1	Introduction	30
2.1.1	Examples of Induction Problems	30
2.1.2	Ockham, Epicurus, Hume, Bayes, Solomonoff	31
2.1.3	Problem Setup	32
2.2	Algorithmic Information Theory	33
2.2.1	Definitions and Notation	33
2.2.2	Turing Machines	34
2.2.3	Kolmogorov Complexity	36
2.2.4	Computability Concepts	38
2.3	Uncertainty & Probabilities	40
2.3.1	Frequency Interpretation: Counting	40
2.3.2	Objective Interpretation: Uncertain Events	41
2.3.3	Subjective Interpretation: Degrees of Belief	43
2.3.4	Determining Priors	45
2.4	Algorithmic Probability & Universal Induction	45
2.4.1	The Universal Prior M	45
2.4.2	Universal Sequence Prediction	47
2.4.3	Universal (Semi)Measures	48
2.4.4	Martin-Löf Randomness	54
2.5	History & References	55
2.6	Problems	60
3	Universal Sequence Prediction	65
3.1	Introduction	67
3.2	Setup and Convergence	68
3.2.1	Random Sequences	68
3.2.2	Universal Prior Probability Distribution	69
3.2.3	Universal Posterior Probability Distribution	70
3.2.4	Convergence of Random Sequences	71
3.2.5	Distance Measures between Probability Distributions	72
3.2.6	Convergence of ξ to μ	74
3.2.7	Convergence in Martin-Löf Sense	76
3.2.8	The Case where $\mu \notin \mathcal{M}$	80
3.2.9	Probability Classes \mathcal{M}	81
3.3	Error Bounds	82
3.3.1	Bayes Optimal Predictors	82
3.3.2	Total Expected Numbers of Errors	82
3.3.3	Proof of Theorem 3.36	84
3.4	Loss Bounds	86

3.4.1	Unit Loss Function	86
3.4.2	Loss Bound of Merhav & Feder	88
3.4.3	Example Loss Functions	89
3.4.4	Proof of Theorem 3.48	89
3.4.5	Convergence of Instantaneous Losses	91
3.4.6	General Loss	92
3.5	Application to Games of Chance	93
3.5.1	Introduction	93
3.5.2	Games of Chance	94
3.5.3	Example	95
3.5.4	Information-Theoretic Interpretation	95
3.6	Optimality Properties	96
3.6.1	Lower Error Bound	96
3.6.2	Pareto Optimality of ξ	99
3.6.3	Balanced Pareto Optimality of ξ	101
3.6.4	On the Optimal Choice of Weights	102
3.6.5	Occam's razor versus No Free Lunches	103
3.7	Miscellaneous	103
3.7.1	Multistep Predictions	104
3.7.2	Continuous Probability Classes \mathcal{M}	106
3.7.3	Further Applications	108
3.7.4	Prediction with Expert Advice	108
3.7.5	Outlook	110
3.8	Summary	111
3.9	Technical Proofs	112
3.9.1	How to Deal with $\mu=0$	112
3.9.2	Entropy Inequalities (Lemma 3.11)	113
3.9.3	Error Inequality (Theorem 3.36)	115
3.9.4	Binary Loss Inequality for $z \leq \frac{1}{2}$ (3.57)	116
3.9.5	Binary Loss Inequality for $z \geq \frac{1}{2}$ (3.58)	117
3.9.6	General Loss Inequality (3.53)	117
3.10	History & References	119
3.11	Problems	119
4	Agents in Known Probabilistic Environments	125
4.1	The AI_μ Model in Functional Form	126
4.1.1	The Cybernetic Agent Model	126
4.1.2	Strings	128
4.1.3	AI Model for Known Deterministic Environment	128
4.1.4	AI Model for Known Prior Probability	130
4.2	The AI_μ Model in Recursive and Iterative Form	132
4.2.1	Probability Distributions	132
4.2.2	Explicit Form of the AI_μ Model	133
4.2.3	Equivalence of Functional and Explicit AI Model	134
4.3	Special Aspects of the AI_μ Model	135

4.3.1	Factorizable Environments	135
4.3.2	Constants and Limits	138
4.3.3	Sequential Decision Theory	139
4.4	Problems	140
5	The Universal Algorithmic Agent AIXI	141
5.1	The Universal AIXI Model	142
5.1.1	Definition of the AIXI Model	142
5.1.2	Universality of M^{AI} and ξ^{AI}	144
5.1.3	Convergence of ξ^{AI} to μ^{AI}	145
5.1.4	Intelligence Order Relation	146
5.2	On the Optimality of AIXI	147
5.3	Value Bounds and Separability Concepts	149
5.3.1	Introduction	149
5.3.2	(Pseudo) Passive μ and the HeavenHell Example	149
5.3.3	The OnlyOne Example	150
5.3.4	Asymptotic Learnability	151
5.3.5	Uniform μ	152
5.3.6	Other Concepts	152
5.3.7	Summary	153
5.4	Value-Related Optimality Results	153
5.4.1	The $\text{AI}\rho$ Models: Preliminaries	153
5.4.2	Pareto Optimality of $\text{AI}\xi$	154
5.4.3	Self-Optimizing Policy p^ξ w.r.t. Average Value	156
5.5	Discounted Future Value Function	159
5.6	Markov Decision Processes (MDP)	165
5.7	The Choice of the Horizon	169
5.8	Outlook	172
5.9	Conclusions	173
5.10	Functions \rightsquigarrow Chronological Semimeasures	173
5.11	Proof of the Entropy Inequality	175
5.12	History & References	177
5.13	Problems	178
6	Important Environmental Classes	185
6.1	Repetition of the $\text{AI}\mu/\xi$ Models	186
6.2	Sequence Prediction (SP)	187
6.2.1	Using the $\text{AI}\mu$ Model for Sequence Prediction	188
6.2.2	Using the $\text{AI}\xi$ Model for Sequence Prediction	190
6.3	Strategic Games (SG)	192
6.3.1	Introduction	192
6.3.2	Strictly Competitive Strategic Games	193
6.3.3	Using the $\text{AI}\mu$ Model for Game Playing	193
6.3.4	Games of Variable Length	195
6.3.5	Using the $\text{AI}\xi$ Model for Game Playing	195

6.4	Function Minimization (FM)	197
6.4.1	Applications/Examples	197
6.4.2	The Greedy Model FMG μ	198
6.4.3	The General FM μ/ξ Model	199
6.4.4	Is the General Model Inventive?	201
6.4.5	Using the AI Models for Function Minimization	202
6.4.6	Remark on TSP	203
6.5	Supervised Learning from Examples (EX)	204
6.5.1	Applications/Examples	204
6.5.2	Supervised Learning with the AI μ/ξ Model	204
6.6	Other Aspects of Intelligence	206
6.7	Problems	207
7	Computational Aspects	209
7.1	The Fastest & Shortest Algorithm for All Problems	210
7.1.1	Introduction & Main Result	210
7.1.2	Levin Search	212
7.1.3	Fast Matrix Multiplication	213
7.1.4	Applicability of the Fast Algorithm M_p^ϵ	214
7.1.5	The Fast Algorithm M_p^ϵ	215
7.1.6	Time Analysis	216
7.1.7	Assumptions on the Machine Model	218
7.1.8	Algorithmic Complexity and the Shortest Algorithm	218
7.1.9	Generalizations	220
7.1.10	Summary & Outlook	220
7.2	Time-Bounded AIXI Model	221
7.2.1	Introduction	221
7.2.2	Time-Limited Probability Distributions	222
7.2.3	The Idea of the Best Vote Algorithm	224
7.2.4	Extended Chronological Programs	224
7.2.5	Valid Approximations	225
7.2.6	Effective Intelligence Order Relation	226
7.2.7	The Universal Time-Bounded AIXI(tl) Agent	226
7.2.8	Limitations and Open Questions	227
7.2.9	Remarks	228
8	Discussion	231
8.1	What has been Achieved	232
8.1.1	Results	232
8.1.2	Comparison to Other Approaches	234
8.2	General Remarks	235
8.2.1	Miscellaneous	235
8.2.2	Prior Knowledge	236
8.2.3	Universal Prior Knowledge	237
8.2.4	How AIXI(tl) Deals with Encrypted Information	237

8.2.5	Mortal Embodied Agents	238
8.3	Personal Remarks	239
8.3.1	On the Foundations of Machine Learning	239
8.3.2	In a World Without Occam	240
8.4	Outlook & Open Questions	241
8.5	Assumptions, Problems, Limitations	242
8.5.1	Assumptions	243
8.5.2	Problems	244
8.5.3	Limitations	244
8.6	Philosophical Issues	245
8.6.1	Turing Test	245
8.6.2	On the Existence of Objective Probabilities	245
8.6.3	Free Will versus Determinism	246
8.6.4	The Big Questions	248
8.7	Conclusions	248
	Bibliography	251
	Index	265

Tables, Figures, Theorems, ...

Table 2.2 ((Prefix) coding of natural numbers and strings)	34
Thesis 2.3 (Turing)	34
Thesis 2.4 (Church)	34
Assumption 2.5 (Short compiler)	34
Definition 2.6 (Prefix/Monotone Turing machine)	35
Theorem 2.7 (Universal prefix/monotone Turing machine)	36
Definition 2.9 (Kolmogorov complexity)	37
Theorem 2.10 (Properties of Kolmogorov complexity)	37
Figure 2.11 (Kolmogorov Complexity)	38
Definition 2.12 (Computable functions)	38
Theorem 2.13 ((Non)computability of Kolmogorov complexity)	39
Axioms 2.14 (Kolmogorov's axioms of probability theory)	41
Definition 2.15 (Conditional probability)	42
Theorem 2.16 (Bayes' rule 1)	42
Axioms 2.17 (Cox's axioms for beliefs)	43
Theorem 2.18 (Cox's theorem)	43
Theorem 2.19 (Bayes' rule 2)	44
Definition 2.22 ((Semi)measures)	46
Theorem 2.23 (Universality of M)	46
Theorem 2.25 (Posterior convergence of M to μ)	48
Theorem 2.28 (Universal (semi)measures)	49
Table 2.29 (Existence of universal (semi)measures)	50
Theorem 2.31 (Martin-Löf random sequences)	54
Definition 2.33 (μ/ξ -random sequences)	54
Definition 3.8 (Convergence of random sequences)	71
Lemma 3.9 (Relations between random convergence criteria)	71
Lemma 3.11 (Entropy inequalities)	72
Theorem 3.19 (Convergence of ξ to μ)	74
Theorem 3.22 (μ/ξ -convergence of ξ to μ)	76
Theorem 3.36 (Error bound)	83
Theorem 3.48 (Unit loss bound)	87
Corollary 3.49 (Unit loss bound)	88
Theorem 3.59 (Instantaneous loss bound)	91
Theorem 3.60 (General loss bound)	92
Theorem 3.63 (Time to win)	94
Theorem 3.64 (Lower error bound)	97

Definition 3.65 (Pareto optimality)	99
Theorem 3.66 (Pareto optimal performance measures)	99
Theorem 3.69 (Balanced Pareto optimality w.r.t. L)	101
Theorem 3.70 (Optimality of universal weights)	102
Theorem 3.74 (Continuous entropy bound)	106
Definition 4.1 (The agent model)	126
Table 4.2 (Notation and emphasis in AI versus control theory)	127
Definition 4.4 (The AI_μ model)	130
Definition 4.5 (The μ /true/generating value function)	130
Figure 4.13 (Expectimax tree/algorithm for $\mathcal{O} = \mathcal{Y} = \mathbb{B}$)	133
Theorem 4.20 (Equivalence of functional and explicit AI model)	134
Theorem 4.25 (Factorizable environments μ)	137
Assumption 4.28 (Finiteness)	138
Claim 5.12 (We expect AIXI to be universally optimal)	146
Definition 5.14 (Intelligence order relation)	147
Definition 5.18 (ρ -Value function)	153
Definition 5.19 (Functional AI_ρ model)	153
Theorem 5.20 (Iterative AI_ρ model)	154
Theorem 5.21 (Linearity and convexity of V_ρ in ρ)	154
Definition 5.22 (Pareto optimal policies)	155
Theorem 5.23 (Pareto optimality of p^ξ)	155
Theorem 5.24 (Balanced Pareto optimality)	155
Lemma 5.27 (Value difference relation)	156
Lemma 5.28 (Convergence of averages)	157
Theorem 5.29 (Self-optimizing policy p^ξ w.r.t. average value)	157
Definition 5.30 (Discounted AI_ρ model and value)	159
Theorem 5.31 (Linearity and convexity of V_ρ in ρ)	160
Theorem 5.32 (Pareto optimality w.r.t. discounted value)	160
Lemma 5.33 (Value difference relation)	160
Theorem 5.34 (Self-optimizing policy p^ξ w.r.t. discounted value)	161
Theorem 5.35 (Continuity of discounted value)	162
Theorem 5.36 (Convergence of universal to true value)	163
Definition 5.37 (Ergodic Markov decision processes)	165
Theorem 5.38 (Self-optimizing policies for ergodic MDPs)	165
Corollary 5.40 (AI_ξ is self-optimizing for ergodic MDPs)	168
Table 5.41 (Effective horizons)	170
Theorem 7.1 (The fastest algorithm)	211
Theorem 7.2 (The fastest & shortest algorithm)	219
Definition 7.8 (Effective intelligence order relation)	226
Theorem 7.9 (Optimality of $AIXItl$)	227
Table 8.1 (Properties of learning algorithms)	234