CONTENTS

Introductory preface 1
How I have (re-)written this book 1
Acknowledgements 2
What I have written in this book 3

I. Smooth Lie group actions on manifolds 9
I.1. Generalities 9
I.2. Equivariant tubular neighborhoods and orbit types decomposition 13
I.3. Examples: $\mathbf{S}^1$-actions on manifolds of dimension 2 and 3 18
I.4. Appendix: Lie groups, Lie algebras, homogeneous spaces 32
Exercises 37

II. Symplectic manifolds 43
II.1. What is a symplectic manifold? 43
II.2. Calibrated almost complex structures 52
II.3. Hamiltonian vector fields and Poisson brackets 58
Exercises 62

III. Symplectic and Hamiltonian group actions 71
III.1. Hamiltonian group actions 71
III.2. Properties of momentum mappings 77
III.3. Torus actions and integrable systems 87
Exercises 97

IV. Morse theory for Hamiltonians 105
IV.1. Critical points of almost periodic Hamiltonians 105
IV.2. Morse functions (in the sense of Bott) 108
IV.3. Connectedness of the fibers of the momentum mapping 111
IV.4. Application to convexity theorems 113
IV.5. Appendix: compact symplectic SU(2)-manifolds of dimension 4 131
Exercises 136

V. Moduli spaces of flat connections 147
V.1. The moduli space of flat connections 147
V.2. A Poisson structure on the moduli space of flat connections 154
V.3. Construction of commuting functions on $\mathcal{M}$ 162
V.4. Appendix: connections on principal bundles 170
Exercises 175

VI. Equivariant cohomology and the Duistermaat–Heckman theorem 177
VI.1. Milnor joins, Borel construction and equivariant cohomology 178
VI.2. Hamiltonian actions and the Duistermaat–Heckman theorem 189
VI.3. Localization at fixed points and the Duistermaat–Heckman formula 201
VI.4. Appendix: some algebraic topology 212
VI.5. Appendix: various notions of Euler classes 218
Exercises 220

VII. Toric manifolds 225
VII.1. Fans and toric varieties 226
VII.2. Symplectic reduction and convex polyhedra 244
VII.3. Cohomology of X_Σ 257
VII.4. Complex toric surfaces 262
Exercises 266

VIII. Hamiltonian circle actions on manifolds of dimension 4 271
VIII.1. Symplectic $\mathbf{S}^1$-actions, generalities 272
VIII.2. Periodic Hamiltonians on 4-dimensional manifolds 279
Exercises 305

Bibliography 311

Index 321