

Contents

Preface	v
Preface to the Second Edition	vii
1 Pricing by Arbitrage	1
1.1 Introduction: Pricing and Hedging	1
1.2 Single-Period Option Pricing Models	10
1.3 A General Single-Period Model	12
1.4 A Single-Period Binomial Model	14
1.5 Multi-period Binomial Models	20
1.6 Bounds on Option Prices	24
2 Martingale Measures	27
2.1 A General Discrete-Time Market Model	27
2.2 Trading Strategies	29
2.3 Martingales and Risk-Neutral Pricing	35
2.4 Arbitrage Pricing: Martingale Measures	38
2.5 Strategies Using Contingent Claims	43
2.6 Example: The Binomial Model	48
2.7 From CRR to Black-Scholes	50
3 The First Fundamental Theorem	57
3.1 The Separating Hyperplane Theorem in \mathbb{R}^n	57
3.2 Construction of Martingale Measures	59
3.3 Pathwise Description	61
3.4 Examples	69
3.5 General Discrete Models	71
4 Complete Markets	87
4.1 Completeness and Martingale Representation	88
4.2 Completeness for Finite Market Models	89
4.3 The CRR Model	91
4.4 The Splitting Index and Completeness	94
4.5 Incomplete Models: The Arbitrage Interval	97
4.6 Characterisation of Complete Models	101

5	Discrete-time American Options	105
5.1	Hedging American Claims	105
5.2	Stopping Times and Stopped Processes	107
5.3	Uniformly Integrable Martingales	110
5.4	Optimal Stopping: The Snell Envelope	116
5.5	Pricing and Hedging American Options	124
5.6	Consumption-Investment Strategies	126
6	Continuous-Time Stochastic Calculus	131
6.1	Continuous-Time Processes	131
6.2	Martingales	135
6.3	Stochastic Integrals	141
6.4	The Itô Calculus	149
6.5	Stochastic Differential Equations	158
6.6	Markov Property of Solutions of SDEs	162
7	Continuous-Time European Options	167
7.1	Dynamics	167
7.2	Girsanov's Theorem	168
7.3	Martingale Representation	174
7.4	Self-Financing Strategies	183
7.5	An Equivalent Martingale Measure	185
7.6	Black-Scholes Prices	193
7.7	Pricing in a Multifactor Model	198
7.8	Barrier Options	204
7.9	The Black-Scholes Equation	214
7.10	The Greeks	217
8	The American Put Option	223
8.1	Extended Trading Strategies	223
8.2	Analysis of American Put Options	226
8.3	The Perpetual Put Option	231
8.4	Early Exercise Premium	234
8.5	Relation to Free Boundary Problems	238
8.6	An Approximate Solution	243
9	Bonds and Term Structure	247
9.1	Market Dynamics	247
9.2	Future Price and Futures Contracts	252
9.3	Changing Numéraire	255
9.4	A General Option Pricing Formula	258
9.5	Term Structure Models	262
9.6	Short-rate Diffusion Models	264
9.7	The Heath-Jarrow-Morton Model	277
9.8	A Markov Chain Model	282

10 Consumption-Investment Strategies	285
10.1 Utility Functions	285
10.2 Admissible Strategies	287
10.3 Maximising Utility of Consumption	291
10.4 Maximisation of Terminal Utility	296
10.5 Consumption and Terminal Wealth	299
11 Measures of Risk	303
11.1 Value at Risk	304
11.2 Coherent Risk Measures	308
11.3 Deviation Measures	316
11.4 Hedging Strategies with Shortfall Risk	320
Bibliography	329
Index	349