

Contents

Preface	xi
Chapter 1 Random Walks A Good Place to Begin	1
1.1. Nearest Neighbor Random Walks on \mathbb{Z}	1
1.1.1. Distribution at Time n	2
1.1.2. Passage Times via the Reflection Principle	3
1.1.3. Some Related Computations	4
1.1.4. Time of First Return	6
1.1.5. Passage Times via Functional Equations	7
1.2. Recurrence Properties of Random Walks	8
1.2.1. Random Walks on \mathbb{Z}^d	9
1.2.2. An Elementary Recurrence Criterion	9
1.2.3. Recurrence of Symmetric Random Walk in \mathbb{Z}^2	11
1.2.4. Transience in \mathbb{Z}^3	13
1.3. Exercises	16
Chapter 2 Doeblin's Theory for Markov Chains	23
2.1. Some Generalities	23
2.1.1. Existence of Markov Chains	24
2.1.2. Transition Probabilities & Probability Vectors	24
2.1.3. Transition Probabilities and Functions	26
2.1.4. The Markov Property	27
2.2. Doeblin's Theory	27
2.2.1. Doeblin's Basic Theorem	28
2.2.2. A Couple of Extensions	30
2.3. Elements of Ergodic Theory	32
2.3.1. The Mean Ergodic Theorem	33
2.3.2. Return Times	34
2.3.3. Identification of π	38
2.4. Exercises	40
Chapter 3 More about the Ergodic Theory of Markov Chains	45
3.1. Classification of States	46
3.1.1. Classification, Recurrence, and Transience	46
3.1.2. Criteria for Recurrence and Transience	48
3.1.3. Periodicity	51
3.2. Ergodic Theory without Doeblin	53
3.2.1. Convergence of Matrices	53

3.2.2. Abel Convergence	55
3.2.3. Structure of Stationary Distributions	57
3.2.4. A Small Improvement	59
3.2.5. The Mean Ergodic Theorem Again	61
3.2.6. A Refinement in The Aperiodic Case	62
3.2.7. Periodic Structure	65
3.3. Exercises	67
Chapter 4 Markov Processes in Continuous Time	75
4.1. Poisson Processes	75
4.1.1. The Simple Poisson Process	75
4.1.2. Compound Poisson Processes on \mathbb{Z}^d	77
4.2. Markov Processes with Bounded Rates	80
4.2.1. Basic Construction	80
4.2.2. The Markov Property	83
4.2.3. The Q -Matrix and Kolmogorov's Backward Equation	85
4.2.4. Kolmogorov's Forward Equation	86
4.2.5. Solving Kolmogorov's Equation	86
4.2.6. A Markov Process from its Infinitesimal Characteristics	88
4.3. Unbounded Rates	89
4.3.1. Explosion	90
4.3.2. Criteria for Non-explosion or Explosion	92
4.3.3. What to Do When Explosion Occurs	94
4.4. Ergodic Properties	95
4.4.1. Classification of States	95
4.4.2. Stationary Measures and Limit Theorems	98
4.4.3. Interpreting $\hat{\pi}_{ii}$	101
4.5. Exercises	102
Chapter 5 Reversible Markov Processes	107
5.1. Reversible Markov Chains	107
5.1.1. Reversibility from Invariance	108
5.1.2. Measurements in Quadratic Mean	108
5.1.3. The Spectral Gap	110
5.1.4. Reversibility and Periodicity	112
5.1.5. Relation to Convergence in Variation	113
5.2. Dirichlet Forms and Estimation of β	115
5.2.1. The Dirichlet Form and Poincaré's Inequality	115
5.2.2. Estimating β_+	117
5.2.3. Estimating β_-	119
5.3. Reversible Markov Processes in Continuous Time	120
5.3.1. Criterion for Reversibility	120
5.3.2. Convergence in $L^2(\hat{\pi})$ for Bounded Rates	121
5.3.3. $L^2(\hat{\pi})$ -Convergence Rate in General	122

5.3.4. Estimating λ	125
5.4. Gibbs States and Glauber Dynamics	126
5.4.1. Formulation	126
5.4.2. The Dirichlet Form	127
5.5. Simulated Annealing	130
5.5.1. The Algorithm	131
5.5.2. Construction of the Transition Probabilities	132
5.5.3. Description of the Markov Process	134
5.5.4. Choosing a Cooling Schedule	134
5.5.5. Small Improvements	137
5.6. Exercises	138
Chapter 6 Some Mild Measure Theory	145
6.1. A Description of Lebesgue's Measure Theory	145
6.1.1. Measure Spaces	145
6.1.2. Some Consequences of Countable Additivity	147
6.1.3. Generating σ -Algebras	148
6.1.4. Measurable Functions	149
6.1.5. Lebesgue Integration	150
6.1.6. Stability Properties of Lebesgue Integration	151
6.1.7. Lebesgue Integration in Countable Spaces	153
6.1.8. Fubini's Theorem	155
6.2. Modeling Probability	157
6.2.1. Modeling Infinitely Many Tosses of a Fair Coin	158
6.3. Independent Random Variables	162
6.3.1. Existence of Lots of Independent Random Variables	163
6.4. Conditional Probabilities and Expectations	165
6.4.1. Conditioning with Respect to Random Variables	166
Notation	167
References	168
Index	169