Contents

1 Background on Continuum Damage Mechanics 1
1.1 Physics and Damage Variables 1
1.1.1 Definition of a Scalar Damage Variable 3
1.1.2 Definition of Several Scalar Damage Variables 3
1.1.3 Definition of a Tensorial Damage Variable 4
1.1.4 Effective Stress Concept 5
1.1.5 Effects of Damage 7
1.2 Thermodynamics of Damage 7
1.2.1 General Framework 7
1.2.2 State Potential for Isotropic Damage 10
1.2.3 State Potential for Anisotropic Damage 11
1.2.4 Quasi-Unilateral Conditions of Microdefects Closure .. 12
1.3 Measurement of Damage 16
1.3.1 Isotropic Elasticity Change 17
1.3.2 Isotropic Elasticity Change by Ultrasonic Waves 17
1.3.3 Anisotropic Elasticity Change 18
1.3.4 Hardness Change 22
1.3.5 Elasticity Field Change 23
1.4 Kinetic Laws of Damage Evolution 26
1.4.1 Damage Threshold and Mesocrack Initiation 27
1.4.2 Formulation of the Isotropic Unified Damage Law 32
1.4.3 Formulation of the Anisotropic Damage Law 34
1.4.4 Fast Identification of Damage Material Parameters ... 35
1.4.5 Generalization of the Unified Damage Law 41
1.5 Elasto-(Visco-)Plasticity Coupled with Damage 45
1.5.1 Basic Equations without Damage Coupling 45
1.5.2 Coupling with Isotropic Damage 52
1.5.3 Coupling with Anisotropic Damage 56
1.5.4 Non-Isothermal Behavior 60
1.5.5 Two-Scale Model for Damage at Microscale 60

1.6 Localization and Mesocrack Initiation 65
1.6.1 Critical Damage Criterion 65
1.6.2 Strain Damage Localization Criterion 65
1.6.3 Size and Orientation of the Crack Initiated 73

2 Numerical Analysis of Damage 77
2.1 Uncoupled Analysis 78
2.1.1 Uniaxial Loading 79
2.1.2 Proportional Loading 82
2.1.3 Post-processing a (Visco-)Plastic Computation 85
2.1.4 Post-processing an Elastic Computation 86
2.1.5 Jump-in-Cycles Procedure in Fatigue 88
2.2 Fully-Coupled Analysis 90
2.2.1 Nonlinear Material Behavior FEA 91
2.2.2 FE Resolution of the Global Equilibrium 94
2.2.3 Local Integration Subroutines 96
2.2.4 Single Implicit Algorithm for Damage Models 97
2.2.5 Damage Models with Microdefects Closure Effect 105
2.2.6 Performing FE Damage Computations 111
2.2.7 Localization Limiters 112
2.3 Locally-Coupled Analysis 114
2.3.1 Post-Processing a Reference Structure Calculation 115
2.3.2 Implicit Scheme for the Two-Scale Model 117
2.3.3 DAMAGE 2000 Post-Processor 119
2.4 Precise Identification of Material Parameters 120
2.4.1 Formulation of an Identification Problem 121
2.4.2 Minimization Algorithm for Least Squares Problems 123
2.4.3 Procedure for Numerical Identification 127
2.4.4 Cross Identification of Damage Evolution Laws 132
2.4.5 Validation Procedure 133
2.4.6 Sensitivity Analysis 135
2.5 Hierarchic Approach and Model Updating 137
2.6 Table of Material Damage Parameters 138

3 Ductile Failures 141
3.1 Engineering Considerations 142
3.2 Fast Calculation of Structural Failures 142
3.2.1 Uniaxial Behavior and Validation of the Damage Law 143
3.2.2 Case of Proportional Loading 144
3.2.3 Sensitivity Analysis 146
3.2.4 Stress Concentration and the Neuber Method 147
3.2.5 Safety Margin and Crack Arrest 152

3.3 Basic Engineering Examples 154
3.3.1 Plates or Members with Holes or Notches 154
3.3.2 Pressurized Shallow Cylinders 156
3.3.3 Post-Buckling in Bending 158
3.3.4 Damage Criteria in Proportional Loading 160
3.4 Numerical Failure Analysis 166
3.4.1 Finite Strains 167
3.4.2 Deep Drawing Limits 170
3.4.3 Damage in Cold Extrusion Process 172
3.4.4 Crack Initiation Direction 174
3.4.5 Porous Materials – the Gurson Model 176
3.4.6 Frames Analysis by Lumped Damage Mechanics 181
3.4.7 Predeformed and Predamaged Initial Conditions 184
3.4.8 Hierarchic Approach up to Full Anisotropy 188

4 Low Cycle Fatigue 191
4.1 Engineering Considerations 192
4.2 Fast Calculation of Structural Failures 192
4.2.1 Uniaxial Behavior and Validation of the Damage Law 192
4.2.2 Case of Proportional Loading 198
4.2.3 Sensitivity Analysis 199
4.2.4 Cyclic Elasto-Plastic Stress Concentration 201
4.2.5 Safety Margin and Crack Growth 208
4.3 Basic Engineering Examples 208
4.3.1 Plate or Members with Holes or Notches 208
4.3.2 Pressurized Shallow Cylinders 210
4.3.3 Cyclic Bending of Beams 212
4.4 Numerical Failure Analysis 213
4.4.1 Effects of Loading History 214
4.4.2 Multiaxial and Multilevel Fatigue Loadings 216
4.4.3 Damage and Fatigue of Elastomers 221
4.4.4 Predeformed and Predamaged Initial Conditions 227
4.4.5 Hierarchic Approach up to Non-Proportional Effects 228

5 Creep, Creep-Fatigue, and Dynamic Failures 233
5.1 Engineering Considerations 234
5.2 Fast Calculation of Structural Failures 234
5.2.1 Uniaxial Behavior and Validation of the Damage Law 235
5.2.2 Case of Proportional Loading 237
5.2.3 Sensitivity Analysis 241
5.2.4 Elasto-Visco-Plastic Stress Concentration 244
5.2.5 Safety Margin and Crack Growth 247

5.3 Basic Engineering Examples 248
5.3.1 Strain Rate and Temperature-Dependent Yield Stress 248
5.3.2 Plates or Members with Holes or Notches 250
5.3.3 Pressurized Shallow Cylinder 253
5.3.4 Adiabatic Dynamics Post-Buckling in Bending 255
5.4 Numerical Failure Analysis 257
5.4.1 Hollow Sphere under External Pressure 258
5.4.2 Effect of Loading History: Creep-Fatigue 262
5.4.3 Creep-Fatigue and Thermomechanical Loadings 263
5.4.4 Dynamic Analysis of Crash Problems 267
5.4.5 Ballistic Impact and Penetration of Projectiles 269
5.4.6 Predeformed and Predamaged Initial Conditions 272
5.4.7 Hierarchic Approach up to Viscous Elastomers 274

6 High Cycle Fatigue 277
6.1 Engineering Considerations 278
6.2 Fast Calculation of Structural Failures 279
6.2.1 Characteristic Effects in High-Cycle Fatigue 279
6.2.2 Fatigue Limit Criteria 281
6.2.3 Two-Scale Damage Model in Proportional Loading 283
6.2.4 Sensitivity Analysis 289
6.2.5 Safety Margin and Crack Growth 292
6.3 Basic Engineering Examples 294
6.3.1 Plates or Members with Holes or Notches 294
6.3.2 Pressurized Shallow Cylinders 295
6.3.3 Bending of Beams 297
6.3.4 Random Loadings 297
6.4 Numerical Failure Analysis 300
6.4.1 Effects of Loading History 300
6.4.2 Non-Proportional Loading of a Thinned Shell 303
6.4.3 Random Distribution of Initial Defects 307
6.4.4 Stochastic Resolution by Monte Carlo Method 309
6.4.5 Predeformed and Predamaged Initial Conditions 312
6.4.6 Hierarchic Approaches up to Surface and Gradient Effects 314

7 Failure of Brittle and Quasi-Brittle Materials 321
7.1 Engineering Considerations 321
7.2 Fast Calculations of Structural Failures 324
7.2.1 Damage Equivalent Stress Criterion 324
7.2.2 Interface Debonding Criterion 327

7.2.3 The Weibull Model 328
7.2.4 Two-Scale Damage Model for Quasi-Brittle Failures 332
7.2.5 Sensitivity Analysis 333
7.2.6 Safety Margin and Crack Propagation 334
7.3 Basic Engineering Examples 336
7.3.1 Plates or Members with Holes and Notches 336
7.3.2 Pressurized Shallow Cylinders 337
7.3.3 Fracture of Beams in Bending 338
7.4 Numerical Failure Analysis 339
7.4.1 Quasi-Brittle Damage Models 339
7.4.2 Failure of Pre-stressed Concrete 3D Structures 346
7.4.3 Seismic Response of Reinforced Concrete Structures 350
7.4.4 Damage and Delamination in Laminate Structures 356
7.4.5 Failure of CMC Structures 361
7.4.6 Single and Multifragmentation of Brittle Materials 364
7.4.7 Hierarchic Approach up to Homogenized Behavior 368

Bibliography 373

Index 375