

# Table of Contents

<b>Preface</b>	ix
<b>Acknowledgements and Sources of Materials</b>	xi
<b>Chapter One: Introduction :</b>	
<b>Optimal Models for Economics and Finance</b>	1
1.1 Introduction	1
1.2 Welfare economics and social choice: modelling and applications	2
1.3 The objectives of this book	5
1.4 An example of an optimal control model	6
1.5 The structure of the book	7
<b>Chapter Two: Mathematics of Optimal Control</b>	9
2.1 Optimization and optimal control models	9
2.2 Outline of the Pontryagin Theory	12
2.3 When is an optimum reached?	14
2.4 Relaxing the convex assumptions	16
2.5 Can there be several optima?	18
2.6 Jump behaviour with a pseudoconcave objective	20
2.7 Generalized duality	24
2.8 Multiobjective (Pareto) optimization	29
2.9 Multiobjective optimal control	30
2.10 Multiobjective Pontryagin conditions	32
<b>Chapter Three: Computing Optimal Control:</b>	
<b>The SCOM package</b>	35
3.1 Formulation and computational approach	35
3.2 Computational requirements	37
3.3 Using the SCOM package	40
3.4 Detailed account of the SCOM package	41
3.4.1 Preamble	41
3.4.2 Format of problem	41
3.4.3 The SCOM codes: The user does not alter them	42
3.5 Functions for the first test problem	46
3.6 The second test problem	47
3.7 The third test problem	49
<b>Chapter Four: Computing Optimal Growth and Development Models</b>	55
4.1 Introduction	55

4.2 The Kendrick-Taylor growth model	56
4.3 The Kendrick-Taylor model implementation	57
4.4 Mathematical and economic properties of the results	60
4.5 Computation by other computer programs	64
4.6 Conclusions	64
<b>Chapter Five: Modelling Financial Investment with Growth</b>	<b>66</b>
5.1 Introduction	66
5.2 Some related literature	66
5.3 Some approaches	69
5.4 A proposed model for interaction between investment and physical capital	70
5.5 A computed model with small stochastic term	72
5.6 Multiple steady states in a dynamic financial model	75
5.7 Sensitivity questions concerning infinite horizons	80
5.8 Some conclusions	81
5.9 The MATLAB codes	82
5.10 The continuity required for stability	83
<b>Chapter Six: Modelling Sustainable Development</b>	<b>84</b>
6.1 Introduction	84
6.2 Welfare measures and models for sustainability	84
6.3 Modelling sustainability	87
6.3.1 Description by objective function with parameters	87
6.3.2 Modified discounting for long-term modelling	89
6.3.3 Infinite horizon model	90
6.4 Approaches that might be computed	92
6.4.1 Computing for a large time horizon	92
6.4.2 The Chichilnisky compared with penalty term model	92
6.4.3 Chichilnisky model compared with penalty model	94
6.4.4 Pareto optimum and intergenerational equality	95
6.4.5 Computing with a modified discount factor	95
6.5 Computation of the Kendrick-Taylor model	96
6.5.1 The Kendrick-Taylor model	96
6.5.2 Extending the Kendrick-Taylor model to include a long time horizon	97
6.5.3 Chichilnisky variant of Kendrick-Taylor model	98
6.5.4 Transformation of the Kendrick-Taylor model	98
6.6 Computer packages and results of computation of models	99
6.6.1 Packages used	99
6.6.2 Results: comparison of the basic model solution with results for modified discount factor	99
6.6.3 Results: effect of increasing the horizon $T$	101

6.6.4 Results: Effect of omitting the growth term in the dynamic equation	103
6.6.5 Results: parametric approach	103
6.6.6 Results: the modified Chichilnisky approach	105
6.7 Existence, uniqueness and global optimization	108
6.8 Conclusions	109
6.9 User programs for transformed Kendrick-Taylor model for sustainable growth	110
<b>Chapter Seven : Modelling and Computing a Stochastic Growth Model</b>	
7.1 Introduction	111
7.2 Modelling stochastic growth	112
7.3 Calculating mean and variance	113
7.4 Computed results for stochastic growth	114
7.5 Requirements for RIOTS_95 as M-files	116
<b>Chapter Eight: Optimization in Welfare Economics</b>	123
8.1 Static and dynamic optimization	123
8.2 Some static welfare models	123
8.3 Perturbations and stability	125
8.4 Some multiobjective optimal control models	126
8.5 Computing multiobjective optima	128
8.6 Some conditions for invexity	129
8.7 Discussion	130
<b>Chapter 9: Transversality Conditions for Infinite Horizon Models</b>	131
9.1 Introduction	131
9.2 Critical literature survey and extensions	131
9.3 Standard optimal control model	135
9.4. Gradient conditions for transversality	136
9.5 The model with infinite horizon	139
9.6 Normalizing a growth model with infinite horizon models	139
9.7 Shadow prices	141
9.8 Sufficiency conditions	142
9.9 Computational approaches for infinite horizon	143
9.10 Optimal control models in finance: special considerations	146
9.11 Conclusions	146
<b>Chapter 10: Conclusions</b>	147
<b>Bibliography</b>	149