

# CONTENTS

<i>Preface</i>	xi
<i>Dependence Among Chapters</i>	xiv
<i>Guide for the Reader</i>	xv
<i>List of Figures</i>	xviii
<i>Acknowledgments</i>	xix
<i>Standard Notation</i>	xxi
<b>I. THEORY OF CALCULUS IN ONE REAL VARIABLE</b>	<b>1</b>
1. Review of Real Numbers, Sequences, Continuity	2
2. Interchange of Limits	13
3. Uniform Convergence	15
4. Riemann Integral	26
5. Complex-Valued Functions	41
6. Taylor's Theorem with Integral Remainder	43
7. Power Series and Special Functions	44
8. Summability	53
9. Weierstrass Approximation Theorem	58
10. Fourier Series	61
11. Problems	78
<b>II. METRIC SPACES</b>	<b>82</b>
1. Definition and Examples	83
2. Open Sets and Closed Sets	91
3. Continuous Functions	95
4. Sequences and Convergence	97
5. Subspaces and Products	102
6. Properties of Metric Spaces	105
7. Compactness and Completeness	108
8. Connectedness	115
9. Baire Category Theorem	117
10. Properties of $C(S)$ for Compact Metric $S$	121
11. Completion	127
12. Problems	130

<b>III. THEORY OF CALCULUS IN SEVERAL REAL VARIABLES</b>	135
1. Operator Norm	135
2. Nonlinear Functions and Differentiation	139
3. Vector-Valued Partial Derivatives and Riemann Integrals	146
4. Exponential of a Matrix	148
5. Partitions of Unity	151
6. Inverse and Implicit Function Theorems	152
7. Definition and Properties of Riemann Integral	161
8. Riemann Integrable Functions	166
9. Fubini's Theorem for the Riemann Integral	169
10. Change of Variables for the Riemann Integral	171
11. Problems	179
<b>IV. THEORY OF ORDINARY DIFFERENTIAL EQUATIONS AND SYSTEMS</b>	183
1. Qualitative Features and Examples	183
2. Existence and Uniqueness	187
3. Dependence on Initial Conditions and Parameters	194
4. Integral Curves	199
5. Linear Equations and Systems, Wronskian	201
6. Homogeneous Equations with Constant Coefficients	208
7. Homogeneous Systems with Constant Coefficients	211
8. Series Solutions in the Second-Order Linear Case	218
9. Problems	226
<b>V. LEBESGUE MEASURE AND ABSTRACT MEASURE THEORY</b>	231
1. Measures and Examples	231
2. Measurable Functions	238
3. Lebesgue Integral	241
4. Properties of the Integral	245
5. Proof of the Extension Theorem	253
6. Completion of a Measure Space	262
7. Fubini's Theorem for the Lebesgue Integral	265
8. Integration of Complex-Valued and Vector-Valued Functions	274
9. $L^1$ , $L^2$ , $L^\infty$ , and Normed Linear Spaces	279
10. Problems	289
<b>VI. MEASURE THEORY FOR EUCLIDEAN SPACE</b>	296
1. Lebesgue Measure and Other Borel Measures	297
2. Convolution	306
3. Borel Measures on Open Sets	314
4. Comparison of Riemann and Lebesgue Integrals	318

<b>VI. MEASURE THEORY FOR EUCLIDEAN SPACE (Continued)</b>	
5. Change of Variables for the Lebesgue Integral	320
6. Hardy–Littlewood Maximal Theorem	327
7. Fourier Series and the Riesz–Fischer Theorem	334
8. Stieltjes Measures on the Line	339
9. Fourier Series and the Dirichlet–Jordan Theorem	346
10. Distribution Functions	350
11. Problems	352
<b>VII. DIFFERENTIATION OF LEBESGUE INTEGRALS ON THE LINE</b>	357
1. Differentiation of Monotone Functions	357
2. Absolute Continuity, Singular Measures, and Lebesgue Decomposition	364
3. Problems	370
<b>VIII. FOURIER TRANSFORM IN EUCLIDEAN SPACE</b>	373
1. Elementary Properties	373
2. Fourier Transform on $L^1$ , Inversion Formula	377
3. Fourier Transform on $L^2$ , Plancherel Formula	381
4. Schwartz Space	384
5. Poisson Summation Formula	389
6. Poisson Integral Formula	392
7. Hilbert Transform	397
8. Problems	404
<b>IX. <math>L^p</math> SPACES</b>	409
1. Inequalities and Completeness	409
2. Convolution Involving $L^p$	417
3. Jordan and Hahn Decompositions	418
4. Radon–Nikodym Theorem	420
5. Continuous Linear Functionals on $L^p$	424
6. Marcinkiewicz Interpolation Theorem	427
7. Problems	436
<b>X. TOPOLOGICAL SPACES</b>	441
1. Open Sets and Constructions of Topologies	441
2. Properties of Topological Spaces	447
3. Compactness and Local Compactness	451
4. Product Spaces and the Tychonoff Product Theorem	458
5. Sequences and Nets	463
6. Quotient Spaces	471

<b>X. TOPOLOGICAL SPACES (Continued)</b>	
7. Urysohn's Lemma	474
8. Metrization in the Separable Case	476
9. Ascoli–Arzelà and Stone–Weierstrass Theorems	477
10. Problems	480
<b>XI. INTEGRATION ON LOCALLY COMPACT SPACES</b>	485
1. Setting	485
2. Riesz Representation Theorem	490
3. Regular Borel Measures	504
4. Dual to Space of Finite Signed Measures	509
5. Problems	517
<b>XII. HILBERT AND BANACH SPACES</b>	520
1. Definitions and Examples	520
2. Geometry of Hilbert Space	526
3. Bounded Linear Operators on Hilbert Spaces	535
4. Hahn–Banach Theorem	537
5. Uniform Boundedness Theorem	543
6. Interior Mapping Principle	545
7. Problems	549
<b>APPENDIX</b>	553
A1. Sets and Functions	553
A2. Mean Value Theorem and Some Consequences	559
A3. Inverse Function Theorem in One Variable	561
A4. Complex Numbers	563
A5. Classical Schwarz Inequality	563
A6. Equivalence Relations	564
A7. Linear Transformations, Matrices, and Determinants	565
A8. Factorization and Roots of Polynomials	568
A9. Partial Orderings and Zorn's Lemma	573
A10. Cardinality	577
<i>Hints for Solutions of Problems</i>	581
<i>Selected References</i>	637
<i>Index of Notation</i>	639
<i>Index</i>	643