
Contents

1	Bases and Basic Sequences	1
1.1	Schauder bases	1
1.2	Examples: Fourier series	6
1.3	Equivalence of bases and basic sequences	10
1.4	Bases and basic sequences: discussion	15
1.5	Constructing basic sequences	19
1.6	The Eberlein-Šmulian Theorem	23
	Problems	25
2	The Classical Sequence Spaces	29
2.1	The isomorphic structure of the ℓ_p -spaces and c_0	29
2.2	Complemented subspaces of ℓ_p ($1 \leq p < \infty$) and c_0	33
2.3	The space ℓ_1	36
2.4	Convergence of series	38
2.5	Complementability of c_0	44
	Problems	48
3	Special Types of Bases	51
3.1	Unconditional bases	51
3.2	Boundedly-complete and shrinking bases	53
3.3	Nonreflexive spaces with unconditional bases	59
3.4	The James space \mathcal{J}	62
3.5	A litmus test for unconditional bases	66
	Problems	69
4	Banach Spaces of Continuous Functions	73
4.1	Basic properties	73
4.2	A characterization of real $\mathcal{C}(K)$ -spaces	75
4.3	Isometrically injective spaces	79
4.4	Spaces of continuous functions on uncountable compact metric spaces	87

4.5	Spaces of continuous functions on countable compact metric spaces	95
	Problems	98
5	$L_1(\mu)$-Spaces and $\mathcal{C}(K)$-Spaces	101
5.1	General remarks about $L_1(\mu)$ -spaces	101
5.2	Weakly compact subsets of $L_1(\mu)$	103
5.3	Weak compactness in $\mathcal{M}(K)$	112
5.4	The Dunford-Pettis property	115
5.5	Weakly compact operators on $\mathcal{C}(K)$ -spaces	118
5.6	Subspaces of $L_1(\mu)$ -spaces and $\mathcal{C}(K)$ -spaces	120
	Problems	122
6	The L_p-Spaces for $1 \leq p < \infty$	125
6.1	Conditional expectations and the Haar basis	125
6.2	Averaging in Banach spaces	131
6.3	Properties of L_1	142
6.4	Subspaces of L_p	148
	Problems	161
7	Factorization Theory	165
7.1	Maurey-Nikishin factorization theorems	165
7.2	Subspaces of L_p for $1 \leq p < 2$	173
7.3	Factoring through Hilbert spaces	180
7.4	The Kwapien-Maurey theorems for type-2 spaces	187
	Problems	191
8	Absolutely Summing Operators	195
8.1	Grothendieck's Inequality	196
8.2	Absolutely summing operators	205
8.3	Absolutely summing operators on $L_1(\mu)$ -spaces	213
	Problems	217
9	Perfectly Homogeneous Bases and Their Applications	221
9.1	Perfectly homogeneous bases	221
9.2	Symmetric bases	227
9.3	Uniqueness of unconditional basis	229
9.4	Complementation of block basic sequences	231
9.5	The existence of conditional bases	235
9.6	Greedy bases	240
	Problems	244

10	ℓ_p-Subspaces of Banach Spaces	247
	10.1 Ramsey theory	247
	10.2 Rosenthal's ℓ_1 theorem	251
	10.3 Tsirelson space	254
	Problems	259
11	Finite Representability of ℓ_p-Spaces	263
	11.1 Finite representability	263
	11.2 The Principle of Local Reflexivity	272
	11.3 Krivine's theorem	275
	Problems	285
12	An Introduction to Local Theory	289
	12.1 The John ellipsoid	289
	12.2 The concentration of measure phenomenon	293
	12.3 Dvoretzky's theorem	296
	12.4 The complemented subspace problem	301
	Problems	306
13	Important Examples of Banach Spaces	309
	13.1 A generalization of the James space	309
	13.2 Constructing Banach spaces via trees	314
	13.3 Pelczyński's universal basis space	316
	13.4 The James tree space	317
A	Fundamental Notions	327
B	Elementary Hilbert Space Theory	331
C	Main Features of Finite-Dimensional Spaces	335
D	Cornerstone Theorems of Functional Analysis	337
	D.1 The Hahn-Banach Theorem	337
	D.2 Baire's Theorem and its consequences	338
E	Convex Sets and Extreme Points	341
F	The Weak Topologies	343
G	Weak Compactness of Sets and Operators	347
	List of Symbols	349
	References	353
	Index	365