

# *Contents*

<b>1. Linear Regression . . . . .</b>	1
1.1 Introduction . . . . .	1
1.2 The Method of Least Squares . . . . .	3
1.2.1 Correlation version . . . . .	7
1.2.2 Large-sample limit . . . . .	8
1.3 The origins of regression . . . . .	9
1.4 Applications of regression . . . . .	11
1.5 The Bivariate Normal Distribution . . . . .	14
1.6 Maximum Likelihood and Least Squares . . . . .	21
1.7 Sums of Squares . . . . .	23
1.8 Two regressors . . . . .	26
Exercises . . . . .	28
<b>2. The Analysis of Variance (ANOVA) . . . . .</b>	33
2.1 The Chi-Square Distribution . . . . .	33
2.2 Change of variable formula and Jacobians . . . . .	36
2.3 The Fisher F-distribution . . . . .	37
2.4 Orthogonality . . . . .	38
2.5 Normal sample mean and sample variance . . . . .	39
2.6 One-Way Analysis of Variance . . . . .	42
2.7 Two-Way ANOVA; No Replications . . . . .	49
2.8 Two-Way ANOVA; Replications and Interaction . . . . .	52
Exercises . . . . .	56
<b>3. Multiple Regression . . . . .</b>	61
3.1 The Normal Equations . . . . .	61

3.2	Solution of the Normal Equations . . . . .	64
3.3	Properties of Least-Squares Estimators . . . . .	70
3.4	Sum-of-Squares Decompositions . . . . .	73
3.4.1	Coefficient of determination . . . . .	79
3.5	Chi-Square Decomposition . . . . .	80
3.5.1	Idempotence, Trace and Rank . . . . .	81
3.5.2	Quadratic forms in normal variates . . . . .	82
3.5.3	Sums of Projections . . . . .	82
3.6	Orthogonal Projections and Pythagoras's Theorem . . . . .	85
3.7	Worked examples . . . . .	89
	Exercises . . . . .	94
<b>4.</b>	<b>Further Multilinear Regression . . . . .</b>	<b>99</b>
4.1	Polynomial Regression . . . . .	99
4.1.1	The Principle of Parsimony . . . . .	102
4.1.2	Orthogonal polynomials . . . . .	103
4.1.3	Packages . . . . .	103
4.2	Analysis of Variance . . . . .	104
4.3	The Multivariate Normal Distribution . . . . .	105
4.4	The Multinormal Density . . . . .	111
4.4.1	Estimation for the multivariate normal . . . . .	113
4.5	Conditioning and Regression . . . . .	115
4.6	Mean-square prediction . . . . .	121
4.7	Generalised least squares and weighted regression . . . . .	123
	Exercises . . . . .	125
<b>5.</b>	<b>Adding additional covariates and the Analysis of Covariance . . . . .</b>	<b>129</b>
5.1	Introducing further explanatory variables . . . . .	129
5.1.1	Orthogonal parameters . . . . .	133
5.2	ANCOVA . . . . .	135
5.2.1	Nested Models . . . . .	139
5.3	Examples . . . . .	140
	Exercises . . . . .	145
<b>6.</b>	<b>Linear Hypotheses . . . . .</b>	<b>149</b>
6.1	Minimisation Under Constraints . . . . .	149
6.2	Sum-of-Squares Decomposition and F-Test . . . . .	152
6.3	Applications: Sequential Methods . . . . .	157
6.3.1	Forward selection . . . . .	157
6.3.2	Backward selection . . . . .	158
6.3.3	Stepwise regression . . . . .	159
	Exercises . . . . .	160

<b>7. Model Checking and Transformation of Data</b> . . . . .	163
7.1 Deviations from Standard Assumptions . . . . .	163
7.2 Transformation of Data . . . . .	168
7.3 Variance-Stabilising Transformations . . . . .	171
7.4 Multicollinearity . . . . .	174
Exercises . . . . .	177
<b>8. Generalised Linear Models</b> . . . . .	181
8.1 Introduction . . . . .	181
8.2 Definitions and examples . . . . .	183
8.2.1 Statistical testing and model comparisons . . . . .	185
8.2.2 Analysis of residuals . . . . .	187
8.2.3 Athletics times . . . . .	188
8.3 Binary models . . . . .	190
8.4 Count data, contingency tables and log-linear models . . . . .	193
8.5 Over-dispersion and the Negative Binomial Distribution . . . . .	197
8.5.1 Practical applications: Analysis of over-dispersed models in R <sup>®</sup> . . . . .	199
Exercises . . . . .	200
<b>9. Other topics</b> . . . . .	203
9.1 Mixed models . . . . .	203
9.1.1 Mixed models and Generalised Least Squares . . . . .	206
9.2 Non-parametric regression . . . . .	211
9.2.1 Kriging . . . . .	213
9.3 Experimental Design . . . . .	215
9.3.1 Optimality criteria . . . . .	215
9.3.2 Incomplete designs . . . . .	216
9.4 Time series . . . . .	219
9.4.1 Cointegration and spurious regression . . . . .	220
9.5 Survival analysis . . . . .	222
9.5.1 Proportional hazards . . . . .	224
9.6 $p >> n$ . . . . .	225
<b>Solutions</b> . . . . .	227
<b>Dramatis Personae: Who did what when</b> . . . . .	269
<b>Bibliography</b> . . . . .	271
<b>Index</b> . . . . .	279