

Contents

ANHA Series Preface	vii
Preface	xv
1 Frames in Finite-dimensional	
Inner Product Spaces	1
1.1 Basic frames theory	2
1.2 Frames in \mathbb{C}^n	12
1.3 The discrete Fourier transform	17
1.4 Pseudo-inverses and the singular value decomposition	20
1.5 Applications in signal transmission	25
1.6 Exercises	30
2 Infinite-dimensional Vector Spaces and Sequences	33
2.1 Normed vector spaces and sequences	33
2.2 Operators on Banach spaces	36
2.3 Hilbert spaces	37
2.4 Operators on Hilbert spaces	38
2.5 The pseudo-inverse operator	40
2.6 A moment problem	42
2.7 The spaces $L^p(\mathbb{R})$, $L^2(\mathbb{R})$, and $\ell^2(\mathbb{N})$	43
2.8 The Fourier transform and convolution	46
2.9 Operators on $L^2(\mathbb{R})$	47
2.10 Exercises	49

3	Bases	51
3.1	Bessel sequences in Hilbert spaces	52
3.2	General bases and orthonormal bases	55
3.3	Riesz bases	59
3.4	The Gram matrix	64
3.5	Fourier series and trigonometric polynomials	69
3.6	Wavelet bases	72
3.7	Bases in Banach spaces	78
3.8	Sampling and analog–digital conversion	83
3.9	Exercises	86
4	Bases and their Limitations	89
4.1	Bases in $L^2(0,1)$ and in general Hilbert spaces	89
4.2	Gabor bases and the Balian–Low Theorem	92
4.3	Bases and wavelets	93
5	Frames in Hilbert Spaces	97
5.1	Frames and their properties	98
5.2	Frames and Riesz bases	105
5.3	Frames and operators	108
5.4	Characterization of frames	112
5.5	Various independency conditions	116
5.6	Perturbation of frames	121
5.7	The dual frames	126
5.8	Continuous frames	129
5.9	Frames and signal processing	130
5.10	Exercises	133
6	B-splines	139
6.1	The B-splines	140
6.2	Symmetric B-splines	146
6.3	Exercises	148
7	Frames of Translates	151
7.1	Frames of translates	152
7.2	The canonical dual frame	162
7.3	Compactly supported generators	165
7.4	Frames of translates and oblique duals	166
7.5	An application to sampling theory	175
7.6	Exercises	176
8	Shift-Invariant Systems	179
8.1	Frame-properties of shift-invariant systems	179
8.2	Representations of the frame operator	191
8.3	Exercises	194

9 Gabor Frames in $L^2(\mathbb{R})$	195
9.1 Basic Gabor frame theory	196
9.2 Tight Gabor frames	210
9.3 The duals of a Gabor frame	212
9.4 Explicit construction of dual frame pairs	216
9.5 Popular Gabor conditions	220
9.6 Representations of the Gabor frame operator and duality	224
9.7 The Zak transform	227
9.8 Time-frequency localization of Gabor expansions	231
9.9 Continuous representations	237
9.10 Exercises	240
10 Gabor Frames in $\ell^2(\mathbb{Z})$	243
10.1 Translation and modulation on $\ell^2(\mathbb{Z})$	243
10.2 Gabor systems in $\ell^2(\mathbb{Z})$ through sampling	244
10.3 Shift-invariant systems	251
10.4 Exercises	252
11 Wavelet Frames in $L^2(\mathbb{R})$	253
11.1 Dyadic wavelet frames	254
11.2 The unitary extension principle	260
11.3 The oblique extension principle	276
11.4 Approximation orders	285
11.5 Construction of pairs of dual wavelet frames	286
11.6 The signal processing perspective	290
11.7 A survey on general wavelet frames	296
11.8 The continuous wavelet transform	300
11.9 Exercises	303
List of Symbols	305
References	307
Index	311