

Contents

Foreword	ix
I Wavelets and Partial Differential Equations	
<i>Silvia Bertoluzza and Silvia Falletta</i>	1
Introduction	3
1 What is a Wavelet?	5
1.1 Multiresolution Analysis	5
1.1.1 Example I: The Haar Basis	13
1.1.2 Example II: B-Splines	14
1.1.3 Example III: Daubechies's Wavelets	14
1.1.4 Example IV: The Schauder Basis	15
1.2 Beyond $L^2(\mathbb{R})$	17
2 The Fundamental Property of Wavelets	23
2.1 The case $\Omega = \mathbb{R}$: The Frequency Domain Point of View vs. The Space Domain Point of View	24
2.2 The General Case: Ω Domain of \mathbb{R}^d	32
2.3 The Issue of Boundary Conditions	35
3 Wavelets for Partial Differential Equations	37
3.1 Wavelet Preconditioning	37
3.2 Nonlinear Wavelet Methods for the Solution of PDEs	41
3.2.1 Nonlinear vs. Linear Wavelet Approximation	41
3.2.2 Nonlinear Solution of PDEs	44
3.3 Wavelet Stabilisation of Unstable Problems	47
3.4 A-Posteriori Error Estimates	50
3.5 Operations on Infinite Matrices and Vectors	51
Bibliography	55

II	High Order Shock-Capturing Schemes for Balance Laws	59
	<i>Giovanni Russo</i>	
1	Introduction	61
1.1	Hyperbolic Systems	62
1.2	Simple Three-Point Schemes	64
1.3	Conservation Form, Jump Conditions and Conservative Schemes	68
1.4	Consistency and Convergence	72
1.5	Conservation Properties	72
1.6	Entropy Condition	73
1.7	Discrete Entropy Condition	75
1.8	Dissipation, Dispersion, and the Modified Equation	75
1.9	Second-Order Methods and Dispersion	77
2	Upwind Scheme for Systems	83
2.1	The Riemann Problem	85
2.2	Godunov Scheme	85
3	The Numerical Flux Function	89
3.1	Higher-Order Extensions of the Godunov Method	89
3.2	The Scalar Equation and Monotone Fluxes	90
4	Nonlinear Reconstruction and High-Order Schemes	97
4.1	High-Order Finite-Volume Schemes	97
4.2	Essentially Non-Oscillatory Reconstruction (ENO)	99
4.3	Weighted ENO Reconstruction (WENO)	102
4.4	Conservative Finite-Difference Schemes	104
4.5	Time Integration: Runge-Kutta Methods	106
4.6	SSP Schemes	107
4.7	Extension to More Dimensions	108
5	Central Schemes	109
5.1	Nessyahu-Tadmor Second-Order Scheme	110
5.2	Description of CRK Schemes	112
5.3	A Second-Order Scheme: CRK2	114
5.4	Higher-Order Schemes: CRK3, CRK4, CRK5	114
5.5	Numerical Tests	118
5.6	Systems of Equations	119
5.7	Componentwise Application	120
5.8	Projection Along Characteristic Directions	123

6	Systems with Stiff Source	125
6.1	Systems of Balance Laws	126
6.2	IMEX Runge-Kutta Schemes	128
6.3	Hyperbolic Systems with Relaxation	130
6.3.1	Zero Relaxation Limit	130
6.3.2	Asymptotic Properties of IMEX Schemes	132
6.4	Numerical Tests	134
6.4.1	Broadwell Model	134
6.4.2	Shallow Water	135
6.4.3	Traffic Flows	137
	Appendix: Butcher Tableau of IMEX-RK	141
	Bibliography	143
III	Discontinuous Galerkin Methods: General Approach and Stability	
	<i>Chi-Wang Shu</i>	149
	Preface	151
1	Introduction	153
2	Time Discretization	155
3	Discontinuous Galerkin Method for Conservation Laws	157
3.1	Two-dimensional Steady-State Linear Equations	157
3.2	One-dimensional Time-dependent Conservation Laws	160
3.2.1	Cell Entropy Inequality and L^2 -Stability	162
3.2.2	Limiters and Total Variation Stability	164
3.2.3	Error Estimates for Smooth Solutions	168
3.3	Comments for Multi-dimensional Cases	170
4	Discontinuous Galerkin Method for Convection-Diffusion Equations	175
4.1	LDG Scheme Formulation	176
4.2	Stability Analysis	177
4.3	Error Estimates	179
4.4	Multi-Dimensions	181
5	Discontinuous Galerkin Method for PDEs Containing Higher-Order Spatial Derivatives	183
5.1	LDG Scheme for the KdV Equations	183
5.1.1	Stability Analysis	185
5.1.2	Error Estimates	187

5.2	LDG Schemes for Other Higher-Order PDEs	190
5.2.1	Bi-harmonic Equations	190
5.2.2	Fifth-Order Convection-Dispersion Equations	191
5.2.3	The $K(m, n)$ Equations	191
5.2.4	The KdV-Burgers-Type (KdVB) Equations	191
5.2.5	The Fifth-Order KdV-Type Equations	192
5.2.6	The Fully Nonlinear $K(n, n, n)$ Equations	192
5.2.7	The Nonlinear Schrödinger (NLS) Equation	192
5.2.8	The Kadomtsev-Petviashvili (KP) Equations	193
5.2.9	The Zakharov-Kuznetsov (ZK) Equation	193
5.2.10	The Kuramoto-Sivashinsky-type Equations	193
5.2.11	The Ito-Type Coupled KdV Equations	194
5.2.12	The Camassa-Holm (CH) Equation	194
5.2.13	The Cahn-Hilliard Equation	194

Bibliography

197