

# Contents

## Geometry of Optimal Control Problems and Hamiltonian Systems

<i>A.A. Agrachev</i> .....	1
1 Lagrange Multipliers' Geometry .....	1
1.1 Smooth Optimal Control Problems .....	1
1.2 Lagrange Multipliers .....	4
1.3 Extremals .....	6
1.4 Hamiltonian System .....	7
1.5 Second Order Information .....	10
1.6 Maslov Index .....	14
1.7 Regular Extremals .....	22
2 Geometry of Jacobi Curves .....	25
2.1 Jacobi Curves .....	25
2.2 The Cross-Ratio .....	26
2.3 Coordinate Setting .....	28
2.4 Curves in the Grassmannian .....	29
2.5 The Curvature .....	30
2.6 Structural Equations .....	33
2.7 Canonical Connection .....	35
2.8 Coordinate Presentation .....	38
2.9 Affine Foliations .....	39
2.10 Symplectic Setting .....	41
2.11 Monotonicity .....	44
2.12 Comparison Theorem .....	49
2.13 Reduction .....	51
2.14 Hyperbolicity .....	53
References .....	58

## Lecture Notes on Logically Switched Dynamical Systems

<i>A.S. Morse</i> .....	61
1 The Quintessential Switched Dynamical System Problem .....	62
1.1 Dwell-Time Switching .....	62

1.2	Switching Between Stabilizing Controllers .....	65
1.3	Switching Between Graphs .....	66
2	Switching Controls with Memoryless Logics .....	67
2.1	Introduction .....	67
2.2	The Problem .....	67
2.3	The Solution .....	67
2.4	Analysis .....	68
3	Collaborations .....	68
4	The Curse of the Continuum .....	69
4.1	Process Model Class .....	69
4.2	Controller Covering Problem .....	73
4.3	A Natural Approach .....	74
4.4	A Different Approach .....	75
4.5	Which Metric? .....	75
4.6	Construction of a Control Cover .....	76
5	Supervisory Control .....	76
5.1	The System .....	77
5.2	Slow Switching .....	86
5.3	Analysis .....	87
5.4	Analysis of the Dwell Time Switching Logic .....	102
6	Flocking .....	110
6.1	Leaderless Coordination .....	111
6.2	Symmetric Neighbor Relations .....	142
6.3	Measurement Delays .....	148
6.4	Asynchronous Flocking .....	155
6.5	Leader Following .....	158
	References .....	159

## **Input to State Stability: Basic Concepts and Results**

	<i>E.D. Sontag</i> .....	163
1	Introduction .....	163
2	ISS as a Notion of Stability of Nonlinear I/O Systems .....	163
2.1	Desirable Properties .....	164
2.2	Merging Two Different Views of Stability .....	165
2.3	Technical Assumptions .....	166
2.4	Comparison Function Formalism .....	166
2.5	Global Asymptotic Stability .....	167
2.6	0-GAS Does Not Guarantee Good Behavior with Respect to Inputs .....	168
2.7	Gains for Linear Systems .....	168
2.8	Nonlinear Coordinate Changes .....	169
2.9	Input-to-State Stability .....	171
2.10	Linear Case, for Comparison .....	172
2.11	Feedback Redesign .....	173
2.12	A Feedback Redesign Theorem for Actuator Disturbances .....	174

3 Equivalences for ISS ..... 176

  3.1 Nonlinear Superposition Principle ..... 176

  3.2 Robust Stability ..... 177

  3.3 Dissipation ..... 178

  3.4 Using “Energy” Estimates Instead of Amplitudes ..... 180

4 Cascade Interconnections ..... 180

  4.1 An Example of Stabilization Using the ISS Cascade Approach .. 182

5 Integral Input-to-State Stability ..... 183

  5.1 Other Mixed Notions ..... 183

  5.2 Dissipation Characterization of iISS ..... 184

  5.3 Superposition Principles for iISS ..... 185

  5.4 Cascades Involving iISS Systems ..... 186

  5.5 An iISS Example ..... 188

6 Input to State Stability with Respect to Input Derivatives ..... 190

  6.1 Cascades Involving the  $D^k$ ISS Property ..... 190

  6.2 Dissipation Characterization of  $D^k$ ISS ..... 191

  6.3 Superposition Principle for  $D^k$ ISS ..... 191

  6.4 A Counter-Example Showing that  $D^1$ ISS  $\neq$  ISS ..... 192

7 Input-to-Output Stability ..... 192

8 Detectability and Observability Notions ..... 194

  8.1 Detectability ..... 195

  8.2 Dualizing ISS to OSS and IOSS ..... 196

  8.3 Lyapunov-Like Characterization of IOSS ..... 196

  8.4 Superposition Principles for IOSS ..... 197

  8.5 Norm-Estimators ..... 197

  8.6 A Remark on Observers and Incremental IOSS ..... 198

  8.7 Variations of IOSS ..... 199

  8.8 Norm-Observability ..... 200

9 The Fundamental Relationship Among ISS, IOS, and IOSS ..... 201

10 Systems with Separate Error and Measurement Outputs ..... 202

  10.1 Input-Measurement-to-Error Stability ..... 202

  10.2 Review: Viscosity Subdifferentials ..... 203

  10.3 RES-Lyapunov Functions ..... 204

11 Output to Input Stability and Minimum-Phase ..... 205

12 Response to Constant and Periodic Inputs ..... 205

13 A Remark Concerning ISS and  $H_\infty$  Gains ..... 206

14 Two Sample Applications ..... 207

15 Additional Discussion and References ..... 209

References ..... 213

**Generalized Differentials, Variational Generators,  
and the Maximum Principle with State Constraints**

*H.J. Sussmann* ..... 221

1 Introduction ..... 221

2	Preliminaries and Background .....	222
2.1	Review of Some Notational Conventions and Definitions .....	222
2.2	Generalized Jacobians, Derivate Containers, and Michel–Penot Subdifferentials .....	228
2.3	Finitely Additive Measures .....	229
3	Cellina Continuously Approximable Maps .....	230
3.1	Definition and Elementary Properties .....	231
3.2	Fixed Point Theorems for CCA Maps .....	234
4	GDQs and AGDQs .....	243
4.1	The Basic Definitions .....	244
4.2	Properties of GDQs and AGDQs .....	246
4.3	The Directional Open Mapping and Transversality Properties .....	255
5	Variational Generators .....	267
5.1	Linearization Error and Weak GDQs .....	267
5.2	GDQ Variational Generators .....	269
5.3	Examples of Variational Generators .....	270
6	Discontinuous Vector Fields .....	277
6.1	Co-Integrably Bounded Integrally Continuous Maps .....	277
6.2	Points of Approximate Continuity .....	280
7	The Maximum Principle .....	281
	References .....	285

**Sliding Mode Control: Mathematical Tools, Design  
and Applications**

<i>V.I. Utkin</i> .....	289	
1	Introduction .....	289
2	Examples of Dynamic Systems with Sliding Modes .....	289
3	VSS in Canonical Space .....	296
3.1	Control of Free Motion .....	298
3.2	Disturbance Rejection .....	300
3.3	Comments for VSS in Canonical Space .....	301
3.4	Preliminary Mathematical Remark .....	302
4	Sliding Modes in Arbitrary State Spaces: Problem Statements .....	303
5	Sliding Mode Equations: Equivalent Control Method .....	305
5.1	Problem Statement .....	305
5.2	Regularization .....	306
5.3	Boundary Layer Regularization .....	311
6	Sliding Mode Existence Conditions .....	313
7	Design Principles .....	316
7.1	Decoupling and Invariance .....	316
7.2	Regular Form .....	318
7.3	Block Control Principle .....	320
7.4	Enforcing Sliding Modes .....	322
7.5	Unit Control .....	325
8	The Chattering Problem .....	327

9	Discrete-Time Systems .....	330
9.1	Discrete-Time Sliding Mode Concept .....	331
9.2	Linear Discrete-Time Systems with Known Parameters.....	333
9.3	Linear Discrete-Time Systems with Unknown Parameters .....	335
10	Infinite-Dimensional Systems .....	336
10.1	Distributed Control of Heat Process .....	337
10.2	Flexible Mechanical System.....	338
11	Control of Induction Motor .....	340
	References .....	344
	<b>List of Participants .....</b>	<b>349</b>