Contents

Fo	reword	• •	 		 •	•	•			•	•	•	•							•	•		xi
1	Introduction		 •	•		•	•	•	•	•	•		•	·	•	•	•	•	•	•	•		1

Part I Pseudoanalytic Function Theory and Second-order Elliptic Equations

2	Defin	Definitions and Results from Bers' Theory						
	2.1	Generating pairs and differentiation	9					
	2.2	Pseudoanalytic functions	11					
	2.3	Derivatives and integrals of pseudoanalytic functions	13					
		2.3.1 Equivalent generating pairs	13					
		2.3.2 Vekua's equation for (F, G) -derivatives	14					
		2.3.3 Integration	16					
3		ions of Second-order Elliptic Equations as Real Components						
		mplex Pseudoanalytic Functions	21					
	3.1	Factorization of the stationary Schrödinger operator						
	3.2	Factorization of the operator $\operatorname{div} p \operatorname{grad} + q$	23					
	3.3	Conjugate metaharmonic functions	27					
	3.4	The main Vekua equation	29					
	3.5	Cauchy's integral theorem for the Schrödinger equation	31					
	3.6	p-analytic functions	32					
4	Form	al Powers						
	4.1	Definition	35					
	4.2	An important special case	37					
	4.3	Similarity principle	38					
	4.4	Taylor series in formal powers	41					
	4.5	The Runge theorem	43					
	4.6	Complete systems of solutions for second-order equations	43					

4.7 A remark on orthogonal coordinate systems in a plane 45



	4.8	Explicit construction of a generating sequence	46
	4.9	Explicit construction of complete systems of solutions	
		of second-order elliptic equations	50
		4.9.1 Explicit construction of complete systems of solutions	
		for a stationary Schrödinger equation	51
		4.9.2 Complete systems of solutions for the	
		${\rm conductivity} \ {\rm equation} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots $	51
	4.10	Numerical solution of boundary value problems	52
5	Cauch	ny's Integral Formula	
	5.1	Preliminary information on the Cauchy integral formula	
		for pseudoanalytic functions	55
	5.2	Relation between the main Vekua equation and the	
		system describing <i>p</i> -analytic functions	57
	5.3	The transplant operator	58
	5.4	Cauchy integral formulas for x^k -analytic functions $\ldots \ldots \ldots$	60
6	Comp	lex Riccati Equation	
	6.1	Preliminary notes	65
	6.2	The two-dimensional stationary Schrödinger equation and	
		the complex Riccati equation	67
	6.3	Generalizations of classical theorems	69
	6.4	Cauchy's integral theorem	71
		· -	

Part II Applications to Sturm-Liouville Theory

7	A Representation for Solutions of the Sturm-Liouville Equation									
	7.1	Solving the one-dimensional Schrödinger equation	75							
	7.2	The "+"-case	76							
	7.3	Two sets of Taylor coefficients	80							
	7.4	Solution of the one-dimensional Schrödinger equation	81							
	7.5	Validating the result	83							
	7.6	The "" case	84							
	7.7	Complex potential	85							
	7.8	Solution of the Sturm-Liouville equation	86							
	7.9	Numerical method for solving Sturm-Liouville equations $\ldots \ldots$	90							
8	Spect	ral Problems and Darboux Transformation								
	8.1	Sturm-Liouville problem as a problem of finding zeros								
		of an analytic function	93							
		8.1.1 Sturm-Liouville problems with spectral parameter								
		dependent boundary conditions	95							

8.2	Numerical method for solving Sturm-Liouville problems	96
8.3	A remark on the Darboux transformation	98

Part III Applications to Real First-order Systems

9	Beltrami Fields										
	9.1	Description of the result	103								
	9.2	Reduction of (9.1) to a Vekua equation	104								
	9.3	Solution in the case when α is a function of one Cartesian variable									
10	Static	Maxwell System in Axially Symmetric Inhomogeneous Media									
	10.1	Meridional and transverse fields	111								
	10.2	Reduction of the static Maxwell system to <i>p</i> -analytic functions	112								
		10.2.1 The meridional case	112								
		10.2.2 The transverse case	112								
	10.3	Construction of formal powers	113								
		10.3.1 Formal powers in the meridional case	113								
		10.3.2 Formal powers in the transverse case	114								

Part IV Hyperbolic Pseudoanalytic Functions

11 Hyperbolic Numbers and Analytic Functions

12	Нуре	bolic Pseudoanalytic Functions	
	12.1	Differential operators	125
	12.2	Hyperbolic pseudoanalytic function theory	126
	12.3	Generating sequences	130
13	and S	onship between Hyperbolic Pseudoanalytic Functions olutions of the Klein-Gordon Equation	100
	13.1	Factorization of the Klein-Gordon equation	133
	13.2	The main hyperbolic Vekua equation	135
	13.3	Generating sequence for the main hyperbolic Vekua equation	

Part V Bicomplex and Biquaternionic Pseudoanalytic Functions and Applications

14	The	Dirac Equation	
	14.1	Notation	150
	14.2	Quaternionic form of the Dirac equation	
	14.3	The Dirac equation in a two-dimensional case as	
		a bicomplex Vekua equation	153
	14.4	Some definitions and results from Bers' theory for	
		bicomplex pseudoanalytic functions	155
		14.4.1 Generating pair, derivative and antiderivative	155
		14.4.2 Generating sequences and Taylor series	
		in formal powers	156
	14.5	The main bicomplex Vekua equation	158
	14.6	Dirac equation with a scalar potential	
15	Com	plex Second-order Elliptic Equations and	
10		mplex Pseudoanalytic Functions	163
16	Mult	idimensional Second-order Equations	
	16.1	Factorization	167
	16.2	The main quaternionic Vekua equation	
Op	en Pi	roblems	171
Bil	bliogr	aphy	173
Inc	dex .		183