

Contents

I	Differential Calculus in the Complex Plane \mathbb{C}	9
I.1	Complex Numbers	9
I.2	Convergent Sequences and Series	24
I.3	Continuity	36
I.4	Complex Derivatives	42
I.5	The CAUCHY-RIEMANN Differential Equations	47
II	Integral Calculus in the Complex Plane \mathbb{C}	69
II.1	Complex Line Integrals	70
II.2	The CAUCHY Integral Theorem	77
II.3	The CAUCHY Integral Formulas	92
III	Sequences and Series of Analytic Functions, the Residue Theorem	103
III.1	Uniform Approximation	104
III.2	Power Series	109
III.3	Mapping Properties of Analytic Functions	124
III.4	Singularities of Analytic Functions	133
III.5	LAURENT Decomposition	142
A	Appendix to III.4 and III.5	155
III.6	The Residue Theorem	162
III.7	Applications of the Residue Theorem	170
IV	Construction of Analytic Functions	191
IV.1	The Gamma Function	192
IV.2	The WEIERSTRASS Product Formula	210
IV.3	The MITTAG-LEFFLER Partial Fraction Decomposition	218
IV.4	The RIEMANN Mapping Theorem	223
A	Appendix : The Homotopical Version of the CAUCHY Integral Theorem	233
B	Appendix : A Homological Version of the CAUCHY Integral Theorem	239

C	Appendix : Characterizations of Elementary Domains	244
V	Elliptic Functions	251
V.1	LIOUVILLE's Theorems	252
A	Appendix to the Definition of the Period Lattice	259
V.2	The WEIERSTRASS \wp -function	261
V.3	The Field of Elliptic Functions	267
A	Appendix to Sect. V.3 : The Torus as an Algebraic Curve .	271
V.4	The Addition Theorem	278
V.5	Elliptic Integrals	284
V.6	ABEL's Theorem	291
V.7	The Elliptic Modular Group	301
V.8	The Modular Function j	309
VI	Elliptic Modular Forms	317
VI.1	The Modular Group and Its Fundamental Region	318
VI.2	The $k/12$ -formula and the Injectivity of the j -function	326
VI.3	The Algebra of Modular Forms	334
VI.4	Modular Forms and Theta Series	338
VI.5	Modular Forms for Congruence Groups	352
A	Appendix to VI.5 : The Theta Group	363
VI.6	A Ring of Theta Functions	370
VII	Analytic Number Theory	381
VII.1	Sums of Four and Eight Squares	382
VII.2	DIRICHLET Series	399
VII.3	DIRICHLET Series with Functional Equations	408
VII.4	The RIEMANN ζ -function and Prime Numbers	421
VII.5	The Analytic Continuation of the ζ -function	429
VII.6	A TAUBERIAN Theorem	436
VIII	Solutions to the Exercises	449
VIII.1	Solutions to the Exercises of Chapter I	449
VIII.2	Solutions to the Exercises of Chapter II	459
VIII.3	Solutions to the Exercises of Chapter III	464
VIII.4	Solutions to the Exercises of Chapter IV	475
VIII.5	Solutions to the Exercises of Chapter V	482
VIII.6	Solutions to the Exercises of Chapter VI	490
VIII.7	Solutions to the Exercises of Chapter VII	498
References		509
Symbolic Notations		519