Contents

1 Introduction 1
1.1 A Few Time Series Concepts 2
1.1.1 Some Simple Stochastic Processes 3
1.1.2 Stationarity, Mean Reversion, Impulse Responses 6
1.1.3 Numerical Exercise: Simulating Simple Stochastic Processes 9
1.2 Structural Macroeconomic Models 12
1.2.1 Static Structural Models 12
1.2.2 Dynamic Structural Models 16
1.2.3 Stochastic, Dynamic Structural Models 21
1.2.4 Stochastic Simulation 23
1.2.5 Numerical Exercise – Simulating Dynamic, Structural Macroeconomic Models 24
1.3 Why are Economic Growth Models Interesting? 27
1.3.1 Microeconomic Foundations of Macroeconomics 27
1.3.2 Lucas' Critique on Economic Policy Evaluation 33
1.3.3 A Brief Overview of Developments on Growth Theory 35
1.3.4 The Use of Growth Models for Actual Policy Making 39
1.4 Numerical Solution Methods 40
1.4.1 Why do we Need to Compute Numerical Solutions to Growth Models? 40
1.4.2 Stability 42
1.4.3 Indeterminacy 43
1.4.4 The Type of Questions We Ask and the Conclusions We Reach 44
1.5 Synopsis of the Book 48

2 The Neoclassical Growth Model Under a Constant Savings Rate 53
2.1 Introduction 53
2.2 Returns to Scale and Sustained Growth 54

Bibliografische Informationen
http://d-nb.info/1000500357

digitalisiert durch DEUTSCHE NATIONAL BIBLIOTHEK

2.3 The Neoclassical Growth Model of Solow and Swan 59
2.3.1 Description of the Model 60
2.3.2 The Dynamics of the Economy 61
2.3.3 Steady-State 64
2.3.4 The Transition Towards Steady-State 68
2.3.5 The Duration of the Transition to Steady-State 69
2.3.6 The Growth Rate of Output and Consumption 69
2.3.7 Convergence in the Neoclassical Model 71
2.3.8 A Special Steady-State: The Golden Rule of Capital Accumulation 73
2.4 Solving the Continuous-Time Solow–Swan Model 76
2.4.1 Solution to the Exact Model 76
2.4.2 The Linear Approximation to the Solow–Swan Model 77
2.4.3 Changes in Structural Parameters 79
2.4.4 Dynamic Inefficiency 82
2.5 The Deterministic, Discrete-Time Solow Swan Model 85
2.5.1 The Exact Solution 85
2.5.2 Approximate Solutions to the Discrete-Time Model 87
2.5.3 Numerical Exercise – Solving the Deterministic Solow–Swan Model 89
2.5.4 Numerical Exercise – A Permanent Change in the Savings Rate 91
2.5.5 Numerical Exercise – Dynamic Inefficiency 93
2.6 The Stochastic, Discrete Time Version of the Solow–Swan Model .. 95
2.6.1 Numerical Exercise – Solving the Stochastic Solow–Swan Model 96
2.7 Exercises 98

3 Optimal Growth. Continuous Time Analysis 101
3.1 The Continuous-Time Version of the Cass–Koopmans Model 101
3.1.1 Optimality Conditions for the Cass–Koopmans Model 103
3.1.2 The Instantaneous Elasticity of Substitution of Consumption (*IES*) 104
3.1.3 Risk Aversion and the Intertemporal Substitution of Consumption 106
3.1.4 Keynes–Ramsey Condition 107
3.1.5 The Optimal Steady-State 108
3.1.6 Numerical Exercise: The Sensitivity of Steady-State Levels to Changes in Structural Parameters 110
3.1.7 Existence, Uniqueness and Stability of Long-Run Equilibrium – A Graphical Discussion 112
3.1.8 Suboptimality of the Golden Rule 114
3.2 Stability and Convergence 115
3.2.1 The Trajectory for Income 119

3.2.2 Numerical Exercise – Characterizing the Transition after a Change in a Structural Parameter 120
3.3 Interpreting the Central Planners's Model as a Competitive Equilibrium Economy 126
3.3.1 The Efficiency of Competitive Equilibrium 129
3.4 A Competitive Equilibrium with Government 131
3.4.1 The Structure of the Economy 131
3.4.2 Feasible Stationary Public Expenditure and Financing Policies 135
3.4.3 Competitive Equilibrium 135
3.4.4 Global Constraint of Resources 136
3.4.5 The Representative Agent Problem 136
3.5 On the Efficiency of Equilibrium with Government 138
3.5.1 On the Efficiency of Equilibrium Under Lump-Sum Taxes and Debt 138
3.5.2 The Inefficiency of the Competitive Equilibrium Allocation Under Distortionary Taxes 140
3.6 The Ricardian Doctrine 146
3.6.1 The Ricardian Doctrine Under Non-Distorting Taxes 146
3.6.2 Failure of the Ricardian Doctrine Under Distorting Taxes ... 147
3.7 Appendix 149
3.7.1 Appendix 1 – Log-linear Approximation to the Continuous Time Version of Cass–Koopmans Model 149
3.7.2 Appendix 2 – An Alternative Presentation of the Equivalence Between the Planner's and the Competitive Equilibrium Mechanisms in an Economy Without Government 150
3.8 Exercises 153

4 Optimal Growth. Discrete Time Analysis 155
4.1 Discrete-Time, Deterministic Cass–Koopmans Model 155
4.1.1 The Global Constraint of Resources 155
4.1.2 Discrete-Time Formulation of the Planner's Problem 157
4.1.3 The Optimal Steady-State 158
4.1.4 The Dynamics of the Model: The Phase Diagram 159
4.1.5 Transversality Condition in Discrete Time 161
4.1.6 Competitive Equilibrium with Government 162
4.2 Fiscal Policy in the Cass–Koopmans Model 167
4.2.1 The Deterministic Case 167
4.2.2 Numerical Exercise – Solving the Deterministic Competitive Equilibrium with Taxes 176
4.2.3 Numerical Exercise – Fiscal Policy Evaluation 179
4.3 Appendices 185
4.3.1 A Reformulation of the Stability Condition for the Deterministic Version of the Model 185

4.3.2 The Intertemporal Government Budget Constraint 187
4.4 Appendix 2: The Ricardian Proposition Under Non-Distortionary Taxes in Discrete Time 190
4.5 Exercises 191

5 Numerical Solution Methods 195
5.1 Numerical Solutions and Simulation Analysis 195
5.2 Analytical Solutions to Simple Growth Models 197
5.2.1 A Model with Full Depreciation 197
5.2.2 A Model with Leisure in the Utility Function 200
5.2.3 Numerical Solutions of the Growth Model Under Full Depreciation 202
5.3 Solving a Simple, Stochastic Version of the Planner's Problem 203
5.3.1 Solving the Linear-Quadratic Approximation to the Planner's Problem 204
5.3.2 The Log-Linear Approximation to the Model 210
5.3.3 The Blanchard–Kahn Solution Method for the Stochastic Planner's Problem. Log-Linear Approximation 212
5.3.4 Uhlig's Undetermined Coefficients Approach. Log-Linear Approximation 215
5.3.5 Sims' Eigenvalue-Eigenvector Decomposition Method Using a Linear Approximation to the Model 217
5.4 Solving the Stochastic Representative Agent's Problem with Taxes 225
5.4.1 The Log-Linear Approximation 227
5.4.2 Numerical Exercise: Solving the Stochastic Representative Agent's Model with Taxes Through Blanchard and Kahn's Approach. Log-Linear Approximation 228
5.4.3 Numerical Exercise: Computing Impulse Responses to a Technology Shock. Log-Linear Approximation 232
5.4.4 Numerical Exercise: Solving the Stochastic Representative Agent's Model with Taxes Through the Eigenvector and Eigenvalue Decomposition Approach. Linear Approximation 234
5.5 Nonlinear Numerical Solution Methods 238
5.5.1 Parameterized Expectations 238
5.5.2 Projection Methods 241
5.6 Appendix – Solving the Planner's Model Under Full Depreciation 251
5.7 Exercises 253

6 Endogenous Growth Models 257
6.1 The *AK* Model 257
6.1.1 Balanced Growth Path 259
6.1.2 Transitional Dynamics 259
6.1.3 Boundedness of Time-Aggregate Utility 261

6.2 The Discrete Time Version of the Model 262
6.2.1 The Transversality Condition and Bounded Utility 265
6.2.2 Absence of Transitional Dynamics: Relationship Between the Stock of Physical Capital and Consumption ... 266
6.3 Stability in the *AK* Model 267
6.4 Effects from Transitory Changes in Policy Parameters 271
6.4.1 A Policy Intervention 272
6.4.2 A Comparison with the Cass–Koopmans Economy 273
6.5 Dynamic Laffer Curves 275
6.5.1 Numerical Exercise on Dynamic Laffer Curves 278
6.6 Solving the Stochastic, Discrete Time Version of the *AK* Model 280
6.6.1 A Linear Approximation to the Stochastic *AK* Model 282
6.6.2 Numerical Exercise: Solving the Stochastic *AK* Model 285
6.7 An Endogenous Growth Model with Productive Public Expenditures: Barro's Model 286
6.8 Transitional Dynamics in Endogenous Growth: The Jones and Manuelli Model 288
6.8.1 Steady-State 290
6.8.2 Solving the Deterministic Version of Jones and Manuelli's Model Through a Linear Approximation 291
6.9 The Stochastic Version of Jones and Manuelli Model 294
6.9.1 Deterministic Balanced Growth Path 295
6.9.2 Transforming the Model in Stationary Ratios 295
6.9.3 The Phase Diagram of the Deterministic Version of the Jones–Manuelli Model: Transitional Dynamics 296
6.9.4 Computing the Dynamics: Log-Linear Approximation 298
6.9.5 Numerical Exercise: Solving the Jones and Manuelli Model 301
6.9.6 The Stochastic *AK* Model as a Special Case 301
6.10 Exercises 302

7 Additional Endogenous Growth Models 305
7.1 Introduction 305
7.2 A Variety of Producer Products 306
7.2.1 The Economy 306
7.2.2 The Inefficiency of the Equilibrium Allocation 314
7.2.3 A Stochastic Version of the Economy with a Variety of Intermediate Goods 316
7.3 Technological Diffusion and Growth 323
7.3.1 The Problem of the Follower Country 324
7.3.2 Deterministic Steady-State 326
7.3.3 Computing the Numerical Solution by Log-Linear Approximations and Numerical Derivatives 328

7.3.4 Numerical Exercise: Solving the Model with Varieties of Intermediate Goods, and the Diffusion Growth Model ... 332
7.4 Schumpeterian Growth ... 333
7.4.1 The Economy ... 334
7.4.2 Computing Equilibrium Trajectories ... 338
7.4.3 Deterministic Steady-State ... 341
7.5 Endogenous Growth with Accumulation of Human Capital ... 342
7.5.1 The Economy ... 343
7.5.2 The Competitive Equilibrium ... 347
7.5.3 Analyzing the Deterministic Steady-State ... 349
7.5.4 Numerical Exercise: Steady-State Effects of Fiscal Policy ... 352
7.5.5 Computing Equilibrium Trajectories in a Stochastic Setup Under the Assumption of Rational Expectations ... 353
7.5.6 Indeterminacy of Equilibria ... 363
7.5.7 Numerical Exercise: The Correlation Between Productivity and Hours Worked in the Human Capital Accumulation Model ... 374
7.6 Exercises ... 376

8 Growth in Monetary Economies: Steady-State Analysis of Monetary Policy ... 377
8.1 Introduction ... 377
8.2 Optimal Growth in a Monetary Economy: The Sidrauski Model ... 378
8.2.1 The Representative Agent's Problem ... 380
8.2.2 Steady-State in the Monetary Growth Economy ... 384
8.2.3 Golden Rule ... 387
8.3 Steady-State Policy Analysis ... 388
8.3.1 Optimal Steady-State Rate of Inflation ... 389
8.3.2 The Welfare Cost of Inflation ... 392
8.4 Two Modelling Issues: Nominal Bonds and the Timing of Real Balances ... 394
8.4.1 Nominal Bonds: The Relationship Between Real and Nominal Interest Rates ... 395
8.4.2 Real Balances in the Utility Function: At the Beginning or at the End of the Period? ... 397
8.4.3 Numerical Exercise: Optimal Rate of Inflation Under Alternative Assumptions on Preferences ... 400
8.5 Monetary Policy Analysis Under Consumption and Income Taxes ... 401
8.5.1 Steady-State ... 403
8.5.2 Numerical Exercise: Computation of Steady-State Levels Under Alternative Policy Choices ... 405
8.6 Monetary Policy Under Endogenous Labor Supply ... 406
8.6.1 The Neutrality of Monetary Policy Under Endogenous Labor Supply ... 406
8.6.2 Numerical Exercise: Evaluation of Steady-State Policies with an Endogenous Labour Supply ... 411

8.7 Optimal Monetary Policy Under Distortionary Taxation and Endogenous Labor 413
8.7.1 The Model 414
8.7.2 Implementability Condition 417
8.7.3 The Ramsey Problem 418
8.8 Exercises 419

9 Transitional Dynamics in Monetary Economies: Numerical Solutions 423
9.1 Introduction 423
9.2 Stability of Public Debt 424
9.3 Alternative Strategies for Monetary Policy: Control of Nominal Rates vs. Money Growth Control 426
9.4 Deterministic Monetary Model with the Monetary Authority Choosing Money Growth 427
9.4.1 Steady-State 429
9.4.2 Solution Through a Log-Linear Approximation 430
9.4.3 Complex Eigenvalues 433
9.5 Deterministic Monetary Model with the Monetary Authority Choosing Nominal Interest Rates 437
9.6 Transitional Effects of Policy Interventions 441
9.6.1 Solving the Model with Nominal Interest Rates as Control Variable, Using a Linear Approximation 442
9.6.2 Numerical Exercise: Changes in Nominal Interest Rates 444
9.6.3 Solving the Model with Money Growth as Control Variable, Using a Linear Approximation 445
9.6.4 Numerical Exercise: Gradual vs. Drastic Changes in Money Growth 448
9.7 The Stochastic Version of the Monetary Model 450
9.7.1 The Monetary Authority Chooses Nominal Interest Rates 452
9.7.2 The Monetary Authority Chooses Money Supply Growth 463
9.8 A New Keynesian Monetary Model 469
9.8.1 A Model Without Capital Accumulation: Ireland's (2004) 470
9.8.2 A New Keynesian Monetary Model with Capital Accumulation 477
9.9 Appendix: In a Log-Linear Approximation, $E_t\hat{\pi}_{t+1} = \hat{i}_t - \hat{r}_t$ 491
9.10 Exercises 492

10 Mathematical Appendix 495
10.1 The Deterministic Control Problem in Continuous Time 495
10.1.1 Transversality Condition 496
10.1.2 The Discounted Problem 496
10.1.3 Calculus of Variations 498
10.2 The Deterministic Control Problem in Discrete Time 499
10.3 First Order Differential Equations 501

10.3.1 1. First Order Differential Equations with Constant Coefficients 501
10.3.2 2. First Order Differential Equations with Variable Coefficients 504
10.4 Matrix Algebra 506
10.4.1 The 2 × 2 Case 508
10.4.2 Systems with a Saddle Path Property 510
10.4.3 Imposing Stability Conditions Over Time 510
10.5 Some Notes on Complex Numbers 513
10.6 Solving a Dynamic Two-Equation System with Complex Roots 514

References 517

Index 521