CONTENTS

Preface ix

1. Overview 1
- 1.0 Chaos, Fractals, and Dynamics 1
- 1.1 Capsule History of Dynamics 2
- 1.2 The Importance of Being Nonlinear 4
- 1.3 A Dynamical View of the World 9

Part I. One-Dimensional Flows

2. Flows on the Line 15
- 2.0 Introduction 15
- 2.1 A Geometric Way of Thinking 16
- 2.2 Fixed Points and Stability 18
- 2.3 Population Growth 21
- 2.4 Linear Stability Analysis 24
- 2.5 Existence and Uniqueness 26
- 2.6 Impossibility of Oscillations 28
- 2.7 Potentials 30
- 2.8 Solving Equations on the Computer 32
- Exercises 36

3. Bifurcations 44
- 3.0 Introduction 44
- 3.1 Saddle-Node Bifurcation 45
- 3.2 Transcritical Bifurcation 50
- 3.3 Laser Threshold 53
- 3.4 Pitchfork Bifurcation 55
- 3.5 Overdamped Bead on a Rotating Hoop 61

3.6 Imperfect Bifurcations and Catastrophes 69
3.7 Insect Outbreak 73
Exercises 79

4. Flows on the Circle 93
4.0 Introduction 93
4.1 Examples and Definitions 93
4.2 Uniform Oscillator 95
4.3 Nonuniform Oscillator 96
4.4 Overdamped Pendulum 101
4.5 Fireflies 103
4.6 Superconducting Josephson Junctions 106
Exercises 113

Part II. Two-Dimensional Flows

5. Linear Systems 123
5.0 Introduction 123
5.1 Definitions and Examples 123
5.2 Classification of Linear Systems 129
5.3 Love Affairs 138
Exercises 140

6. Phase Plane 145
6.0 Introduction 145
6.1 Phase Portraits 145
6.2 Existence, Uniqueness, and Topological Consequences 148
6.3 Fixed Points and Linearization 150
6.4 Rabbits versus Sheep 155
6.5 Conservative Systems 159
6.6 Reversible Systems 163
6.7 Pendulum 168
6.8 Index Theory 174
Exercises 181

7. Limit Cycles 196
7.0 Introduction 196
7.1 Examples 197
7.2 Ruling Out Closed Orbits 199
7.3 Poincaré-Bendixson Theorem 203
7.4 Liénard Systems 210
7.5 Relaxation Oscillators 211
7.6 Weakly Nonlinear Oscillators 215
Exercises 227

8. Bifurcations Revisited 241
8.0 Introduction 241
8.1 Saddle-Node, Transcritical, and Pitchfork Bifurcations 241
8.2 Hopf Bifurcations 248
8.3 Oscillating Chemical Reactions 254
8.4 Global Bifurcations of Cycles 260
8.5 Hysteresis in the Driven Pendulum and Josephson Junction 265
8.6 Coupled Oscillators and Quasiperiodicity 273
8.7 Poincaré Maps 278
Exercises 284

Part III. Chaos

9. Lorenz Equations 301
9.0 Introduction 301
9.1 A Chaotic Waterwheel 302
9.2 Simple Properties of the Lorenz Equations 311
9.3 Chaos on a Strange Attractor 317
9.4 Lorenz Map 326
9.5 Exploring Parameter Space 330
9.6 Using Chaos to Send Secret Messages 335
Exercises 341

10. One-Dimensional Maps 348
10.0 Introduction 348
10.1 Fixed Points and Cobwebs 349
10.2 Logistic Map: Numerics 353
10.3 Logistic Map: Analysis 357
10.4 Periodic Windows 361
10.5 Liapunov Exponent 366
10.6 Universality and Experiments 369
10.7 Renormalization 379
Exercises 388

11. Fractals 398
11.0 Introduction 398
11.1 Countable and Uncountable Sets 399
11.2 Cantor Set 401
11.3 Dimension of Self-Similar Fractals 404
11.4 Box Dimension 409
11.5 Pointwise and Correlation Dimensions 411
Exercises 416

12. Strange Attractors 423
12.0 Introduction 423
12.1 The Simplest Examples 423
12.2 Hénon Map 429
12.3 Rössler System 434
12.4 Chemical Chaos and Attractor Reconstruction 437
12.5 Forced Double-Well Oscillator 441
Exercises 448

Answers to Selected Exercises 455
References 465
Author Index 475
Subject Index 478