

# Contents

<i>Preface</i>	<i>page</i>	<i>ix</i>
<b>1</b>	<b>Complex Numbers</b>	<b>1</b>
1.1	Complex numbers and their representations	1
1.2	Algebraic properties of complex numbers	5
1.2.1	De Moivre's theorem	7
1.3	Geometric properties of complex numbers	13
1.3.1	$n$ th roots of unity	16
1.3.2	Symmetry with respect to a circle	17
1.4	Some topological definitions	22
1.5	Complex infinity and the Riemann sphere	26
1.5.1	The Riemann sphere and stereographic projection	27
1.6	Applications to electrical circuits	29
1.7	Problems	33
<b>2</b>	<b>Analytic Functions</b>	<b>40</b>
2.1	Functions of a complex variable	40
2.1.1	Velocity of fluid flow emanating from a source	42
2.1.2	Mapping properties of complex functions	44
2.1.3	Definitions of exponential and trigonometric functions	47
2.2	Limit and continuity of complex functions	48
2.2.1	Limit of a complex function	48
2.2.2	Continuity of a complex function	50
2.3	Differentiation of complex functions	52
2.3.1	Complex velocity and acceleration	54
2.4	Cauchy-Riemann relations	55
2.4.1	Conjugate complex variables	60
2.5	Analyticity	61

2.6	Harmonic functions	63
2.6.1	Harmonic conjugate	65
2.6.2	Steady state temperature distribution	69
2.6.3	Poisson's equation	73
2.7	Problems	74
<b>3</b>	<b>Exponential, Logarithmic and Trigonometric Functions</b>	<b>81</b>
3.1	Exponential functions	81
3.1.1	Definition from the first principles	82
3.1.2	Mapping properties of the complex exponential function	84
3.2	Trigonometric and hyperbolic functions	85
3.2.1	Mapping properties of the complex sine function	90
3.3	Logarithmic functions	92
3.3.1	Heat source	94
3.3.2	Temperature distribution in the upper half-plane	96
3.4	Inverse trigonometric and hyperbolic functions	99
3.5	Generalized exponential and power functions	102
3.6	Branch points, branch cuts and Riemann surfaces	105
3.6.1	Joukowski mapping	109
3.7	Problems	112
<b>4</b>	<b>Complex Integration</b>	<b>118</b>
4.1	Formulations of complex integration	118
4.1.1	Definite integral of a complex-valued function of a real variable	119
4.1.2	Complex integrals as line integrals	120
4.2	Cauchy's integral theorem	127
4.3	Cauchy's integral formula and its consequences	136
4.3.1	Derivatives of contour integrals	138
4.3.2	Morera's theorem	142
4.3.3	Consequences of the Cauchy integral formula	142
4.4	Potential functions of conservative fields	147
4.4.1	Velocity potential and stream function of fluid flows	147
4.4.2	Electrostatic fields	160
4.4.3	Gravitational fields	164
4.5	Problems	169
<b>5</b>	<b>Taylor and Laurent Series</b>	<b>177</b>
5.1	Complex sequences and series	177
5.1.1	Convergence of complex sequences	177

5.1.2	Infinite series of complex numbers	179
5.1.3	Convergence tests of complex series	180
5.2	Sequences and series of complex functions	183
5.2.1	Convergence of series of complex functions	184
5.2.2	Power series	186
5.3	Taylor series	190
5.4	Laurent series	196
5.4.1	Potential flow past an obstacle	205
5.5	Analytic continuation	208
5.5.1	Reflection principle	211
5.6	Problems	212
<b>6</b>	<b>Singularities and Calculus of Residues</b>	<b>221</b>
6.1	Classification of singular points	221
6.2	Residues and the Residue Theorem	227
6.2.1	Computational formulas for evaluating residues	228
6.3	Evaluation of real integrals by residue calculus	232
6.3.1	Integrals of trigonometric functions over $[0, 2\pi]$	232
6.3.2	Integrals of rational functions	234
6.3.3	Integrals involving multi-valued functions	236
6.3.4	Miscellaneous types of integral	239
6.4	Fourier transforms	243
6.4.1	Fourier inversion formula	244
6.4.2	Evaluation of Fourier integrals	248
6.5	Cauchy principal value of an improper integral	251
6.6	Hydrodynamics in potential fluid flows	258
6.6.1	Blasius laws of hydrodynamic force and moment	258
6.6.2	Kutta–Joukowski’s lifting force theorem	262
6.7	Problems	263
<b>7</b>	<b>Boundary Value Problems and Initial-Boundary Value Problems</b>	<b>272</b>
7.1	Integral formulas of harmonic functions	273
7.1.1	Poisson integral formula	273
7.1.2	Schwarz integral formula	280
7.1.3	Neumann problems	285
7.2	The Laplace transform and its inversion	287
7.2.1	Bromwich integrals	291
7.3	Initial-boundary value problems	298
7.3.1	Heat conduction	298
7.3.2	Longitudinal oscillations of an elastic thin rod	303
7.4	Problems	308

<b>8</b>	<b>Conformal Mappings and Applications</b>	319
8.1	Conformal mappings	319
8.1.1	Invariance of the Laplace equation	326
8.1.2	Hodograph transformations	332
8.2	Bilinear transformations	335
8.2.1	Circle-preserving property	338
8.2.2	Symmetry-preserving property	341
8.2.3	Some special bilinear transformations	350
8.3	Schwarz–Christoffel transformations	358
8.4	Problems	368
	<i>Answers to Problems</i>	377
	<i>Index</i>	389