

Contents

Preface	ix
1 Introduction	1
2 Background Material on Domain Decomposition	
2.1 Introduction	9
2.2 Domain Decomposition for 1-d Problems	10
2.2.1 Unbounded Domains	10
2.2.2 Bounded Domains	14
2.2.3 Semi-discretization	14
2.3 Domain Decomposition Methods for Elliptic Problems	17
2.3.1 Review of Basic Methods	17
2.3.2 Virtual Controls	27
2.3.3 The Basic Algorithm of P.-L. Lions	40
2.3.4 An Augmented Lagrangian Formulation	46
2.3.5 General Elliptic Problems and More General Splittings	48
2.3.6 An a Posteriori Error Estimate	54
2.3.7 Interpretation as a Damped Richardson iteration	57
2.3.8 A Serial One-Dimensional Problem	64
3 Partial Differential Equations on Graphs	
3.1 Introduction	71
3.2 Partial Differential Operators on Graphs	72
3.3 Elliptic Problems on Graphs	76
3.3.2 Domain Decomposition	79
3.3.3 Convergence	82
3.3.4 Interpretation as a Richardson Iteration	86
3.4 Hyperbolic Problems on Graphs	91
3.4.1 The Model	91
3.4.2 The Domain Decomposition Procedure	96

4 Optimal Control of Elliptic Problems	
4.1 Introduction	107
4.2 Distributed Controls	109
4.2.2 Domain Decomposition	111
4.2.3 A Complex Helmholtz Problem and its Decomposition	111
4.2.4 Convergence	113
4.2.5 Methods for Elliptic Optimal Control Problems	116
4.2.6 An A Posteriori Error Estimate	117
4.3 Boundary Controls	120
4.3.2 Domain Decomposition	120
4.3.3 Convergence	121
4.3.4 An A Posteriori Error Estimate	125
5 Control of Partial Differential Equations on Graphs.	
5.1 Introduction	131
5.2 Elliptic Problems	131
5.2.1 The Global Optimal Control Problem on a Graph	131
5.2.2 Domain Decomposition	133
5.2.3 Distributed Controls	135
5.2.4 Boundary Controls	141
5.3 Hyperbolic Problems	146
5.3.1 The Global Optimal Control Problem on a Graph	146
5.3.2 The Domain Decomposition Procedure	148
6 Control of Dissipative Wave Equations	
6.1 Introduction	159
6.2 Optimal Dissipative Boundary Control	160
6.2.1 Setting the Problem	160
6.2.2 Existence and Regularity of Solutions	162
6.2.3 The Global Optimality System	172
6.3 Time Domain Decomposition	173
6.3.1 Description of the Algorithm	173
6.3.2 Convergence of the Iterates	179
6.3.3 A Posteriori Error Estimates	188
6.3.4 Extension to General Dissipative Control Systems	201
6.4 Decomposition of the Spatial Domain	208
6.4.1 Description of the Algorithm	208
6.4.2 Convergence of the Iterates	216
6.4.3 A Posteriori Error Estimates	231
6.5 Space and Time Domain Decomposition	239
6.5.1 Sequential Space-Time Domain Decomposition	239
6.5.2 Sequential Time-Space Domain Decomposition	250

7 Boundary Control of Maxwell's System	
7.1 Introduction	257
7.2 Optimal Dissipative Boundary Control	258
7.2.1 Setting the Problem	258
7.2.2 Existence and Uniqueness of Solution	260
7.2.3 The Global Optimality System	265
7.3 Time Domain Decomposition	266
7.3.1 Description of the Algorithm	266
7.3.2 Convergence of the Iterates	270
7.3.3 A Posteriori Error Estimates	277
7.4 Decomposition of the Spatial Domain	289
7.4.1 Description of the Algorithm	289
7.4.2 Convergence of the Iterates	293
7.4.3 A Posteriori Error Estimates	306
7.5 Time and Space Domain Decomposition	313
7.5.1 Sequential Space-Time Domain Decomposition	313
7.5.2 Sequential Time-Space Domain Decomposition	317
8 Control of Conservative Wave Equations	
8.1 Introduction	321
8.2 Optimal Boundary Control	322
8.2.1 Setting the Problem	322
8.2.2 Existence and Regularity of Solutions	324
8.2.3 The Global Optimality System	326
8.3 Time Domain Decomposition	327
8.3.1 Description of the Algorithm	327
8.3.2 Convergence of the Iterates	329
8.3.3 A Posteriori Error Estimates	333
8.3.4 Extension to General Conservative Control Systems	337
8.4 Decomposition of the Spatial Domain	340
8.4.1 The Local Optimality Systems	340
8.4.2 The Domain Decomposition Algorithm	342
8.4.3 Convergence of the Iterates	345
8.5 The Exact Reachability Problem	353
8.5.1 The Global Optimality System	353
8.5.2 The Limit of the Local Optimality Systems	355
8.5.3 Application to Domain Decomposition	365
8.5.4 Convergence to the Global Optimality System	369

9 Domain Decomposition for 2-D Networks	
9.1 Elliptic Systems on 2-D Networks	375
9.1.2 Examples	378
9.1.3 Existence and Uniqueness of Solutions	384
9.1.4 Domain Decomposition	386
9.1.5 Convergence of the Algorithm	388
9.2 Optimal Control on 2-D Networks	395
9.2.1 Optimal Final Value Control	395
9.2.2 Existence and Regularity of Solutions	397
9.3 Decomposition of the Spatial Domain	400
9.3.2 The Decomposition Algorithm	403
9.3.3 Convergence of the Algorithm	409
Bibliography	435
Index	441