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Corrections to this book

A list of misprints and corrections to this book is posted on the World-Wide Web at the URL <http://www.hep.psi.ch/denner/GTupdates.html>, or can be obtained by writing to the authors. We would be grateful, if you would report additional errors in the book to Ansgar.Denner@psi.ch.