
Contents

Preface	ix
Chapter 1. Introduction	1
§1.1. Basic Definitions and Examples	1
§1.2. The Category of Quiver Representations	3
§1.3. Representation Spaces	6
§1.4. Indecomposable Representations	8
§1.5. The Path Algebra	11
§1.6. Duality	14
§1.7. The Krull-Remak-Schmidt Theorem	15
§1.8. Bibliographical Remarks	17
Chapter 2. Homological Algebra of Quiver Representations	19
§2.1. Projective and Injective Modules	19
§2.2. Projective and Injective Quiver Representations	22
§2.3. The Hereditary Property of Path Algebras	24
§2.4. The Extensions Group	27
§2.5. The Euler Form	32
§2.6. Bibliographical Remarks	34
Chapter 3. Finite Dimensional Algebras	35
§3.1. Quivers with Relations	35
§3.2. The Jacobson Radical	38
§3.3. Basic Algebras	41

§3.4. Morita Equivalence	43
§3.5. Bibliographical Remarks	47
Chapter 4. Gabriel's Theorem	49
§4.1. Quivers of Finite Representation Type	50
§4.2. Dynkin Graphs	52
§4.3. The Reflection Functors	57
§4.4. The Coxeter Functor	64
§4.5. Examples	69
§4.6. Bibliographical Remarks	71
Chapter 5. Almost Split Sequences	73
§5.1. Ideals of Morphisms in the Module Categories	73
§5.2. Irreducible Morphisms	77
§5.3. The Auslander-Reiten Quiver	83
§5.4. The Notion of an Almost Split Sequence	86
§5.5. Bibliographical Remarks	94
Chapter 6. Auslander-Reiten Theory	97
§6.1. Injective Envelopes and Projective Covers	97
§6.2. The Transpose Functor	100
§6.3. The Translation Functor for Quivers	102
§6.4. Auslander-Reiten Duality	103
§6.5. Coxeter Functors Revisited	107
§6.6. The Auslander-Reiten Quiver for Hereditary Algebras	111
§6.7. The Preprojective Algebra	114
§6.8. Bibliographical Remarks	116
Chapter 7. Extended Dynkin Quivers	117
§7.1. Representations of the Kronecker Quiver	118
§7.2. The Auslander-Reiten Quiver of the Kronecker Quiver	121
§7.3. AR Quivers for other Extended Dynkin Types	122
§7.4. Bibliographical Remarks	129
Chapter 8. Kac's Theorem	131
§8.1. Deformed Preprojective Algebras	131
§8.2. Reflections	136
§8.3. Root Systems	138

§8.4. Quiver Representations over Finite Fields	142
§8.5. Bibliographical Remarks	147
Chapter 9. Geometric Invariant Theory	149
§9.1. Algebraic Group Actions	150
§9.2. Linearly Reductive Groups	155
§9.3. The Geometry of Quotients	162
§9.4. Semi-Invariants and the Sato-Kimura Lemma	164
§9.5. Geometric Invariant Theory	167
§9.6. The Hilbert-Mumford Criterion	169
§9.7. GIT for Quiver Representations	172
§9.8. GIT Quotients with Respect to Weights	176
§9.9. Bibliographical Remarks	182
Chapter 10. Semi-invariants of Quiver Representations	183
§10.1. Background from Classical Invariant Theory	184
§10.2. The Le Bruyn-Procesi Theorem	187
§10.3. Background from the Representation Theory of GL_n	191
§10.4. Semi-invariants and Representation Theory	197
§10.5. Examples for Dynkin Quivers	199
§10.6. Schofield Semi-invariants	204
§10.7. The Main Theorem and Saturation Theorem	206
§10.8. Proof of the Main Theorem	211
§10.9. Semi-invariants for Dynkin Quivers	216
§10.10. Semi-invariants for Extended Dynkin Types	218
§10.11. More Examples of Rings of Semi-invariants	225
§10.12. Schofield Incidence Varieties	231
§10.13. Bibliographical Remarks	240
Chapter 11. Orthogonal Categories and Exceptional Sequences	243
§11.1. Schur Representations	244
§11.2. The Canonical Decomposition	246
§11.3. Tilting Modules	254
§11.4. Orthogonal Categories	259
§11.5. Quivers with Two Vertices	266
§11.6. Two Sincerity Results	269
§11.7. The Braid Group Action on Exceptional Sequences	270

§11.8. Examples	273
§11.9. An Algorithm for the Canonical Decomposition	275
§11.10. Bibliographical Remarks	285
Chapter 12. Cluster Categories	287
§12.1. A Combinatorial Model for Type A_n	288
§12.2. Cluster Combinatorics and Decorated Representations	294
§12.3. Triangulated Categories and Derived Categories	303
§12.4. The Derived Category of Quiver Representations	310
§12.5. Cluster Categories	316
§12.6. Cluster Tilted Algebras	318
§12.7. Bibliographical Remarks	322
Notation	325
Index	327
Bibliography	331