

# Contents

<b>Preface</b>	xiii
Notation	xvii
<b>PART I FUNDAMENTALS OF GEOPHYSICAL FLUID DYNAMICS</b>	<b>1</b>
<b>1 Equations of Motion</b>	<b>3</b>
1.1 Time Derivatives for Fluids	3
1.2 The Mass Continuity Equation	7
1.3 The Momentum Equation	11
1.4 The Equation of State	13
1.5 Thermodynamic Relations	14
1.6 Thermodynamic Equations for Fluids	21
1.7 Thermodynamics of Seawater	30
1.8 Sound Waves	40
1.9 Compressible and Incompressible Flow	41
1.10 The Energy Budget	42
1.11 An Introduction to nondimensionalization and Scaling	46
Appendix A: Thermodynamics of an Ideal gas from the Gibbs function	47
Appendix B: The First Law of Thermodynamics for Fluids	49
<b>2 Effects of Rotation and Stratification</b>	<b>55</b>
2.1 Equations of Motion in a Rotating Frame	55
2.2 Equations of Motion in Spherical Coordinates	59
2.3 Cartesian Approximations: The Tangent Plane	69
2.4 The Boussinesq Approximation	70
2.5 The Anelastic Approximation	75
2.6 Pressure and other Vertical Coordinates	79
2.7 Scaling for Hydrostatic Balance	83
2.8 Geostrophic and Thermal Wind Balance	87
2.9 Gradient Wind Balance	94
2.10 Static Instability and the Parcel Method	97
Appendix A: Asymptotic Derivation of the Boussinesq Equations	101

<b>3</b>	<b>Shallow Water Systems</b>	<b>105</b>
3.1	Dynamics of a Single Shallow Layer of Fluid	105
3.2	Reduced Gravity Equations	110
3.3	Multi-Layer Shallow Water Equations	112
3.4	From Continuous Stratification to Shallow Water	114
3.5	Geostrophic Balance and Thermal Wind	118
3.6	Form Stress	119
3.7	Conservation Properties of Shallow Water Systems	120
3.8	Shallow Water Waves	123
3.9	Geostrophic Adjustment	127
3.10	Isentropic Coordinates	134
3.11	Available Potential Energy	137
<b>4</b>	<b>Vorticity and Potential Vorticity</b>	<b>143</b>
4.1	Vorticity and Circulation	143
4.2	The Vorticity Equation	145
4.3	Vorticity and Circulation Theorems	147
4.4	Vorticity Equation in a Rotating Frame	153
4.5	Potential Vorticity Conservation	156
4.6	Potential Vorticity in the Shallow Water System	162
4.7	Potential Vorticity in Approximate, Stratified Models	163
4.8	The Impermeability of Isentropes to Potential Vorticity	165
<b>5</b>	<b>Geostrophic Theory</b>	<b>171</b>
5.1	Geostrophic Scaling	171
5.2	The Planetary-Geostrophic Equations	176
5.3	The Shallow Water Quasi-Geostrophic Equations	180
5.4	The Continuously Stratified Quasi-Geostrophic System	187
5.5	Quasi-Geostrophy and Ertel Potential Vorticity	195
5.6	Energetics of Quasi-Geostrophy	198
5.7	The Ekman Layer	201
<b>PART II</b>	<b>WAVES, INSTABILITIES AND TURBULENCE</b>	<b>213</b>
<b>6</b>	<b>Wave Fundamentals</b>	<b>215</b>
6.1	Fundamentals and Formalities	215
6.2	Group Velocity	220
6.3	Ray Theory	224
6.4	Rossby Waves	226
6.5	Rossby Waves in Stratified Quasi-Geostrophic Flow	231
6.6	Energy Propagation and Reflection of Rossby Waves	234
6.7	Group Velocity, Revisited	240
6.8	Energy Propagation of Poincaré Waves	244
	Appendix A: The wKB Approximation for Linear Waves	247
<b>7</b>	<b>Gravity Waves</b>	<b>251</b>
7.1	Surface Gravity Waves	251
7.2	Shallow Water Waves on Fluid Interfaces	257
7.3	Internal Waves in a Continuously Stratified Fluid	259
7.4	Internal Wave Reflection	268

7.5	Internal Waves in a Fluid with Varying Stratification	271
7.6	Internal Waves in a Rotating Frame of Reference	276
7.7	Topographic Generation of Internal Waves	283
7.8	Acoustic-Gravity Waves in an Ideal Gas	293
<b>8</b>	<b>Linear Dynamics at Low Latitudes</b>	<b>297</b>
8.1	Co-existence of Rossby and Gravity Waves	298
8.2	Waves on the Equatorial Beta Plane	303
8.3	Ray Tracing and Equatorial Trapping	314
8.4	Forced-Dissipative Wavelike Flow	316
8.5	Forced, Steady Flow: the Matsuno–Gill Problem	321
	Appendix A: Nondimensionalization and Parabolic Cylinder Functions	330
	Appendix B: Mathematical Relations in the Matsuno–Gill Problem	333
<b>9</b>	<b>Barotropic and Baroclinic Instability</b>	<b>335</b>
9.1	Kelvin–Helmholtz Instability	335
9.2	Instability of Parallel Shear Flow	337
9.3	Necessary Conditions for Instability	345
9.4	Baroclinic Instability	347
9.5	The Eady Problem	351
9.6	Two-Layer Baroclinic Instability	356
9.7	A Kinematic View of Baroclinic Instability	363
9.8	The Energetics of Linear Baroclinic Instability	367
9.9	Beta, Shear and Stratification in a Continuous Model	369
<b>10</b>	<b>Waves, Mean-Flows, and their Interaction</b>	<b>379</b>
10.1	Quasi-Geostrophic Wave–Mean-Flow Interaction	380
10.2	The Eliassen–Palm Flux	383
10.3	The Transformed Eulerian Mean	387
10.4	The Non-Acceleration Result	394
10.5	Influence of Eddies on the Mean-Flow in the Eady Problem	399
10.6	Necessary Conditions for Instability	403
10.7	Necessary Conditions for Instability: Use of Pseudoenergy	406
<b>11</b>	<b>Basics of Incompressible Turbulence</b>	<b>413</b>
11.1	The Fundamental Problem of Turbulence	413
11.2	The Kolmogorov Theory	416
11.3	Two-dimensional Turbulence	423
11.4	Predictability of Turbulence	433
11.5	Spectra of Passive Tracers	437
<b>12</b>	<b>Geostrophic Turbulence and Baroclinic Eddies</b>	<b>445</b>
12.1	Differential Rotation in Two-dimensional Turbulence	445
12.2	Stratified Geostrophic Turbulence	454
12.3	A Scaling Theory for Geostrophic Turbulence	460
12.4	Phenomenology of Baroclinic Eddies in the Atmosphere and Ocean	464

<b>13</b>	<b>Turbulent Diffusion and Eddy Transport</b>	<b>473</b>
13.1	Diffusive Transport	473
13.2	Turbulent Diffusion	475
13.3	Two-Particle Diffusivity	480
13.4	Mixing Length Theory	484
13.5	Homogenization of a Scalar that is Advected and Diffused	487
13.6	Diffusive Fluxes and Skew Fluxes	490
13.7	Eddy Diffusion in the Atmosphere and Ocean	493
13.8	Thickness and Potential Vorticity Diffusion	502
<b>PART III</b>	<b>LARGE-SCALE ATMOSPHERIC CIRCULATION</b>	<b>509</b>
<b>14</b>	<b>The Overturning Circulation: Hadley and Ferrel Cells</b>	<b>511</b>
14.1	Basic Features of the Atmosphere	511
14.2	A Steady Model of the Hadley Cell	516
14.3	A Shallow Water Model of the Hadley Cell	524
14.4	Asymmetry Around the Equator	525
14.5	Eddy Effects on the Hadley Cell	528
14.6	Non-local Eddy Effects and Numerical Results	532
14.7	The Ferrel Cell	534
<b>15</b>	<b>Zonally-Averaged Mid-Latitude Atmospheric Circulation</b>	<b>539</b>
15.1	Surface Westerlies and the Maintenance of a Barotropic Jet	540
15.2	Layered Models of the Mid-Latitude Circulation	549
15.3	Eddy Fluxes and an Example of a Closed Model	562
15.4	A Stratified Model and the Real Atmosphere	566
15.5	Tropopause Height and the Stratification of the Troposphere	572
15.6	A Model for both Stratification and Tropopause Height	579
	Appendix A: TEM for the Primitive Equations in Spherical Coordinates	581
<b>16</b>	<b>Planetary Waves and Zonal Asymmetries</b>	<b>585</b>
16.1	Rossby Wave Propagation in a Slowly Varying Medium	585
16.2	Horizontal Propagation of Rossby Waves	588
16.3	Critical Lines and Critical Layers	594
16.4	A wkb Wave–Mean-Flow Problem for Rossby Waves	598
16.5	Vertical Propagation of Rossby waves	599
16.6	Vertical Propagation of Rossby Waves in Shear	606
16.7	Forced and Stationary Rossby Waves	609
16.8	Effects of Thermal Forcing	615
16.9	Wave Propagation Using Ray Theory	621
<b>17</b>	<b>The Stratosphere</b>	<b>627</b>
17.1	A Descriptive Overview	627
17.2	Waves in the Stratosphere	634
17.3	Wave Momentum Transport and Deposition	639
17.4	Phenomenology of the Residual Overturning Circulation	642
17.5	Dynamics of the Residual Overturning Circulation	644
17.6	The Quasi-Biennial Oscillation	652
17.7	Variability and Extra-Tropical Wave–Mean-Flow Interaction	663

<b>18</b>	<b>Water Vapour and the Tropical Atmosphere</b>	<b>673</b>
18.1	A Moist Ideal Gas	673
18.2	The Distribution of Relative Humidity	680
18.3	Atmospheric Convection	691
18.4	Convection in a Moist Atmosphere	695
18.5	Radiative Equilibrium	700
18.6	Radiative-Convective Equilibrium	703
18.7	Vertically-Constrained Equations of Motion for Large Scales	708
18.8	Scaling and Balanced Dynamics for Large-Scale Flow in the Tropics	711
18.9	Scaling and Balance for Large-Scale Flow with Diabatic Sources	714
18.10	Convectively Coupled Gravity Waves and the MJO	717
	Appendix A: Moist Thermodynamics from the Gibbs Function	720
	Appendix B: Equations of Radiative Transfer	724
	Appendix C: Analytic Approximation of Tropopause Height	725
<b>PART IV</b>	<b>LARGE-SCALE OCEANIC CIRCULATION</b>	<b>729</b>
<b>19</b>	<b>Wind-Driven Gyres</b>	<b>731</b>
19.1	The Depth Integrated Wind-Driven Circulation	733
19.2	Using Viscosity Instead of Drag	740
19.3	Zonal Boundary Layers	744
19.4	The Nonlinear Problem	745
19.5	Inertial Solutions	747
19.6	Topographic Effects on Western Boundary Currents	753
<b>20</b>	<b>Structure of the Upper Ocean</b>	<b>761</b>
20.1	Vertical Structure of the Wind-Driven Circulation	761
20.2	A Model with Continuous Stratification	767
20.3	Observations of Potential Vorticity	770
20.4	The Main Thermocline	774
20.5	Scaling and Simple Dynamics of the Main Thermocline	776
20.6	The Internal Thermocline	779
20.7	The Ventilated Thermocline	785
	Appendix A: Miscellaneous Relationships in a Layered Model	796
<b>21</b>	<b>The Meridional Overturning Circulation and the ACC</b>	<b>801</b>
21.1	Sideways Convection	802
21.2	The Maintenance of Sideways Convection	808
21.3	Simple Box Models	813
21.4	A Laboratory Model of the Abyssal Circulation	818
21.5	A Model for Oceanic Abyssal Flow	821
21.6	A Model of Deep Wind-Driven Overturning	829
21.7	The Antarctic Circumpolar Current	836
21.8	A Dynamical Model of the Residual Overturning Circulation	845
21.9	A Model of the Interhemispheric Circulation	853

<b>22</b>	<b>Equatorial Circulation and El Niño</b>	<b>861</b>
22.1	Observational Preliminaries	861
22.2	Dynamical Preliminaries	862
22.3	A Local Model of the Equatorial Undercurrent	865
22.4	An Ideal Fluid Model of the Equatorial Undercurrent	876
22.5	An Introduction to El Niño and the Southern Oscillation	886
22.6	The Walker Circulation	891
22.7	The Oceanic Response	893
22.8	Coupled Models and Unstable Interactions	895
22.9	Simple Conceptual and Numerical Models of ENSO	898
22.10	Numerical Solutions of the Shallow Water Equations	902
	Appendix A: Derivation of a Delayed-Oscillator Model	904
	<b>References</b>	<b>909</b>
	<b>Index</b>	<b>936</b>

In the main text, sections that are more advanced or that contain material that is peripheral to the main narrative are marked with a black diamond,  $\blacklozenge$ . Sections that contain material that is still not settled or that describe active areas of research are marked with a dagger,  $\dagger$ .