

Crystals, Defects and Microstructures
Modeling Across Scales

ROB PHILLIPS

Brown University, Providence RI

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