

# Contents

Chapter 1. Introduction	1
1. Sudakov's strategy in the strictly convex case	2
2. Sudakov's strategy in the general convex case	5
3. Structure of the paper	18
Chapter 2. General notations and definitions	21
1. Functions and multifunctions	21
2. Affine subspaces, convex sets and norms	22
3. Measures and disintegration	24
4. Optimal transportation problems	26
5. Linear preorders, uniqueness and optimality	27
6. Optimal transportation problems with convex norm and cone costs	33
7. Transportation problems with convex norms and cone costs on Lipschitz graphs	34
8. Optimal transportation problems on directed locally affine partitions	35
9. From directed partitions to directed fibrations	41
Chapter 3. Directed locally affine partitions on cone-Lipschitz foliations	49
1. Convex cone-Lipschitz graphs	49
2. Convex cone-Lipschitz foliations	51
3. Regular transport sets and residual set	54
4. Super/subdifferential directed partitions of regular sets	60
5. Analysis of the residual set	63
6. Optimal transportation on $c_{\tilde{C}}$ -Lipschitz foliations	65
7. Dimensional reduction on directed partitions via cone approximation property	69
8. Model sets of directed segments	69
9. $k$ -dimensional model sets	74
10. $k$ -dimensional sheaf sets and $\mathcal{D}$ -cylinders	76
11. Negligibility of initial/final points	77
Chapter 4. Proof of Theorem 1.1	81
Chapter 5. From $\tilde{C}^k$ -fibrations to linearly ordered $\tilde{C}^k$ -Lipschitz foliations	85
1. Construction of a $(c_{\tilde{C}^k}, \tilde{\mu}, \tilde{\nu})$ -compatible linear preorder	86
2. Minimal $(c_{\tilde{C}^k}, \tilde{\mu}, \tilde{\nu})$ -compatible linear preorder	91
3. Cone approximation property for linearly ordered $c_{\tilde{C}}$ -Lipschitz foliations	93
Chapter 6. Proof of Theorems 1.2-1.6.	101

1. Proof of Theorems 1.6 and 1.2	101
2. Proof of Theorems 1.3 and 1.5	102
Appendix A. Minimality of equivalence relations	103
Chapter B. Notation	105
Chapter C. Index of definitions	109
Bibliography	111