

Contents

Chapter 0. Introduction	1
Basic properties	3
Embedding	4
Characterizations	4
Interpolation	6
Multipliers	6
Chapter 1. Preliminaries	9
1.1. Noncommutative L_p -spaces	9
1.2. Quantum tori	10
1.3. Fourier multipliers	12
1.4. Hardy spaces	14
Chapter 2. Sobolev spaces	19
2.1. Distributions on quantum tori	19
2.2. Definitions and basic properties	21
2.3. A Poincaré-type inequality	25
2.4. Lipschitz classes	28
2.5. The link with the classical Sobolev spaces	31
Chapter 3. Besov spaces	35
3.1. Definitions and basic properties	35
3.2. A general characterization	42
3.3. The characterizations by Poisson and heat semigroups	48
3.4. The characterization by differences	51
3.5. Limits of Besov norms	54
3.6. The link with the classical Besov spaces	55
Chapter 4. Triebel-Lizorkin spaces	59
4.1. A multiplier theorem	59
4.2. Definitions and basic properties	68
4.3. A general characterization	72
4.4. Concrete characterizations	76
4.5. Operator-valued Triebel-Lizorkin spaces	80
Chapter 5. Interpolation	83
5.1. Interpolation of Besov and Sobolev spaces	83
5.2. The K-functional of (L_p, W_p^k)	88
5.3. Interpolation of Triebel-Lizorkin spaces	91

Chapter 6. Embedding	93
6.1. Embedding of Besov spaces	93
6.2. Embedding of Sobolev spaces	95
6.3. Compact embedding	100
Chapter 7. Fourier multiplier	103
7.1. Fourier multipliers on Sobolev spaces	103
7.2. Fourier multipliers on Besov spaces	107
7.3. Fourier multipliers on Triebel-Lizorkin spaces	110
Acknowledgements	113
Bibliography	115